Question 1

Let ‘Jim is inside’ be ‘p’ and ‘Jan is at the pool’ be ‘q’.
Then the statement becomes \( p \land q \)
The negation of \( p \land q \) is \(~( p \land q )\)
Using de Morgans Law \(~( p \land q )\equiv ~ p \lor ~ q\)
So substituting for ‘p’ and ‘q’, the negation of the statement is:
‘Jim is not inside or Jan is not at the pool’

So the correct answer is D.

Question 2

\( w \equiv ‘Mary is wealthy’ \)
\( f \equiv ‘Mary does not have good friends’ \)
\( i \equiv ‘Mary is intelligent’ \)
\( l \equiv ‘Mary is lucky’ \)

The symbolic statement \(~f \land ( w \land i ) \rightarrow l\)
translates to: ‘If Mary has good friends and is wealthy and intelligent then Mary is lucky’.

(a) ‘If Mary has good friends and is wealthy and is intelligent, then Mary is lucky’ becomes \(~f \land w \land i \rightarrow l\)
However \(~f \land w \land i \rightarrow l\equiv ~f \land ( w \land i ) \rightarrow l\) (Associative Law)
So the statement is equivalent to the symbolic statement.

(b) ‘If Mary is not lucky, then Mary does not have good friends or is not wealthy or is not intelligent’ becomes
\(~l \rightarrow f \lor ~w \lor ~i\)
\equiv ~l \rightarrow f \lor (~w \lor ~i)\) (Associative Law)
\equiv ~l \rightarrow f \lor (~w \lor i)\) ( de Morgans Law)
\equiv ~f \land (~w \land i) \rightarrow l\) ( Contrapositive )
\equiv ~f \land ~(~w \land i) \rightarrow l\) ( double negative)
\equiv ~f \land ( w \land i) \rightarrow l\) ( double negative )

So the statement is equivalent to the symbolic statement

(c) \( f \lor ~w \lor ~i \lor l\equiv ~f \lor ~w \lor ~i \lor ~l\) ( double negative )
\equiv ~(~f \lor ~w \lor ~i \lor ~l)\) ( de Morgans law )
\equiv ~(~f \land w \land i \lor ~l)\) ( double negative )
\equiv ~(~f \land ( w \land i ) \lor ~l)\) ( Associative law )
\equiv ~(~f \land ( w \land i ) \lor ~l)\) ( negation )

The negation of ‘\( p \rightarrow q \)’ is logically equivalent to ‘\( p \land ~q \)’
The statement is equivalent to the symbolic statement.

(d) \( l \rightarrow ~f \land w \land i\) is the converse of the conditional statement.

The conditional statement and its converse are not logically equivalent.

The correct answer to the question is D.

Question 3

WRT the set of real numbers \( \mathbb{R} \) and the set of integers \( \mathbb{Z} \), the sets
\( D_1 = \{-4, -3, -2, -1, 0\} \)
\( D_2 = \{ 0, 1, 2, 3, 4 \} \)

(i) For all \( m \in D_1 \) there exists \( n \in D_2 \) such that \( m = -n \)
If \( m = -4, n = 4 \)
\( m = -3, n = 3 \)
\( m = -2, n = 2 \)
\( m = -1, n = 1 \)
\( m = 0, n = 0 \)
\( \therefore \) the statement is true for all \( m \) and \( n \) in \( D_1 \) and \( D_2 \) respectively.

(ii) There exists \( n \in D_2 \) for all \( m \in D_1 \) such that \( m + n = m \)
Here \( n = 0. \) for all \( m \in D_1 \) such that \( m + n = m. \) i.e. \( m + 0 = m. \)
\( \therefore \) the statement is true.

(iii) For all \( m \in Z \) there exists \( n \in Z \) such that \( m < -n. \)
The statement is true for \( m \leq 0 \) and \( n < 0. \)
If \( m \geq 0 \) and \( n < 0 \) the statement is only true for \( |m| < |n| \)
If \( m \leq 0 \) and \( n > 0 \) the statement is only true for \( |m| > |n| \)
If \( n = 0 \) the statement is only true for \( m < 0. \)
If \( m \geq 0 \) and \( n > 0 \) the statement is not true for any \( m, n \)
\( \therefore \) the statement is not true.

(iv) For all \( n \in R \land n \neq 0 \) there exists \( m \in R \) such that \( m \cdot n = 1. \)
If \( m = \frac{1}{n} \) then \( m \cdot n = 1 \) so the statement is true.

The solution to the question is \( A ( \text{true, true, false, true} ) \)

**Question 4**

Let \( p \) be ‘I have a debt to the University’ and \( q \) be ‘I cannot get my exam results’
This translates symbolically to: \( p \rightarrow q \)

The negation of the statement is: \(~(p \rightarrow q) \equiv p \land \sim q\)
which when substituted becomes: “I have a debt to the University and I can get my exam results.”

The inverse of the statement is: \(~ p \rightarrow \sim q\)
which when substituted becomes: “If I do not have a debt to the University then I can get my exam results.”

\( \therefore \) The correct answer to the question is \( B \)
Question 5

(i) \( p \rightarrow q \) is false only if \( p \) is true and \( q \) is false.

Truth Table for \( p \rightarrow q \)

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\[ \therefore \text{the statement is correct.} \]

(ii) From Page 31 of the text the definition of syllogism is an argument form consisting of 2 premises and one conclusion.

\[ \therefore \text{the statement is incorrect.} \]

(iii) From Page 32 of text the definition of inferencing is that ‘The validity of modus tollens can be shown to follow from modus ponens together with the fact that a conditional statement is logically equivalent to its contrapositive.’

Modus ponens: \( p \rightarrow q \), \( p \), \( \therefore q \)
Contrapositive \( \sim q \rightarrow \sim p \)
Modus Tollens \( p \rightarrow q \), \( \sim q \), \( \therefore \sim p \)

So the statement is correct.

(iv) The definition of the symbol \( \leftrightarrow \) is given on Page 84 of the text. The notation means that every element of the first set must correspond to an element in the second set and vice-versa. The statement is incorrect because the two sets need to completely overlap. \( P(x) \) and \( Q(x) \) have to have identical truth sets.

The left diagram shows \( P(x) \) and \( Q(x) \) overlapping to a certain degree and the right diagram shows \( P(x) \Leftrightarrow Q(x) \).

\[ \text{The statement is not correct.} \]

(v) The predicate of a statement is a part of a statement where the subject has been removed. The predicate tells us information about the subject. (Text Page 75, 76). A predicate is a sentence where some or all nouns are replaced by variables. The sentence becomes a statement when specific values are substituted for the variables.

\[ \text{So the statement is incorrect.} \]

The correct solution to the question is part A.
Question 6

A Let $p$ be ‘Kevin Rudd is the present PM’ and let $q$ be ‘He is a politician’.
This is an example of modus ponens.
$p \rightarrow q$
$p$
$\therefore q$
Statement is valid.

B Let $p$ be ‘You are a discrete Maths student’ and $q$ be ‘You are good at logical thinking’
$p \rightarrow q$
$q$
$\therefore p$
This statement is invalid.
Truth table for $p \rightarrow q$

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This shows that $q$ can occur without $p$.

C Let ‘The exchange rate WRT the US dollar is going up’ be $p$ and ‘The country’s economy is slowing down’ be $q$.
i.e. $p \rightarrow q$
$\sim p$
$\therefore \sim q$
The table above shows that this argument is also invalid.

D Let $p$ be ‘I get high marks for this quiz’ and $q$ be ‘I won’t get low marks for the exam’ and $r$ be ‘I will get a present from my dad’.
$p \rightarrow q$
$q \rightarrow r$
$r \rightarrow p$

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The argument is not valid. A conclusion cannot be used to make assumptions about the premise of an argument.
The only valid argument is A.
Question 7

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<th>p\lor q</th>
<th>\neg p\lor q</th>
<th>(p\lor q)/(\neg p\lor q)</th>
<th>(p\lor q)/(\neg p\lor q) \leftrightarrow q</th>
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The correct solution is C.

Question 8

(i) A conditional statement is logically equivalent to its contrapositive. The converse and inverse of a statement are logically equivalent to each other but not to the conditional statement.
∴ the statement is not correct.

(ii) Universal modus ponens:
for every $x$  
$P(x) \rightarrow Q(x)$  
for particular $x$  
$P(a)$  
∴ $Q(a)$

Here $x \equiv$ car, $a \equiv$ Omnex
$P(x) \equiv$ no good car
$Q(x) \equiv$ is cheap

$P(x) \rightarrow Q(x)$(No good car is cheap)
$P(a)$ (Omnex is not a good car)
∴ $Q(a)$ (Omnex is cheap)
∴ the statement is correct.

(iii) See Page 111 of the text.
The statement is correct.

(iv) Let the statement “infinite series converges” $\equiv p$
Let the statement “terms of the infinite series $\frac{1}{n}$ go to 0” $\equiv q$

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The conclusion of an argument cannot be used to draw conclusions about the premise of an argument.
The statement is incorrect.

(v) The two’s complement of 101010012 is (01010110 + 1)2. This is 010101112
or $(1+2^1+2^2+2^4+2^6)_{10} = (1 + 2 + 4 + 16 + 64)_{10} = 87_{10}$
So the statement is valid.

The correct solution is B (ii), (iii) and (v) are correct).
Question 9

$11001110_2 + 3CE_{16} = 11001110_2 + 001111001110_2$

$= 010010011100_2$

$= 2234_{16}$

$= 49C_{16}$

$= (2^1 + 2^2 + 2^3 + 2^6 + 2^7 + 2^8 + 2^9)_{10}$

$= (2+4+64+128+256+512)_{10} = 974_{10}$

The solution is A.

Question 10

The statement is: ‘There exists integers $a, b, c$, and $d$ such that $a^4 + b^4 + c^4 = d^4$’

$Z$ is the set of integers.

$\exists \equiv$ there exists

The statement becomes

$\exists (a,b,c,d) \in Z$ such that $a^4 + b^4 + c^4 = d^4$

$\therefore$ The correct answer is D.

Question 11

The statement in symbolic terms is

$\forall r \in \mathbb{R}$ if $(r^2 - 1) \geq 0$ then $r > 0$.

Let “$(r^2 - 1) \geq 0$” be $p$ and “$r > 0$” be $q$.

$p \rightarrow q$  

The contrapositive is $\neg q \rightarrow \neg p$

or  

If $r \leq 0$ then $(r^2 - 1) < 0$

The converse of the statement is $q \rightarrow p$

or  

If $r > 0$ then $(r^2 - 1) \geq 0$

The inverse of the statement is $\neg p \rightarrow \neg q$

or  

If $(r^2 - 1) < 0$ then $r \leq 0$

The correct answer is then C

$\forall r \in \mathbb{R}$ If $r \leq 0$ then $(r^2 - 1) < 0$

$\forall r \in \mathbb{R}$ if $r > 0$ then $(r^2 - 1) \geq 0$

$\forall r \in \mathbb{R}$ if $(r^2 - 1) < 0$ then $r \leq 0$. 
Question 12

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<tr>
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<th>(~ P ∧ ~ Q)</th>
<th>(P ∧ Q) ∨ (~ P ∨ Q)</th>
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(i) The Boolean expression for the digital circuit is \((P \land Q) \lor (~ P \land ~ Q)\)  
This is equivalent to the original Boolean expression.

(ii) The Boolean expression is \((~ P \lor Q) \land (P \lor ~ Q)\).

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<tr>
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<th>(P ∨ ~ Q)</th>
<th>(~P ∨ Q) \land (P ∨ ~Q)</th>
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The expression is logically equivalent to the original expression.

(iii) The Boolean expression is \((P \land Q) \lor ~ (P \lor Q)\)  
Here we use de Morgan’s Law  
\((P \land Q) \lor ~ (P \lor Q) \equiv (P \land Q) \lor (~P \lor ~Q)\)  
This is equivalent to the original expression.

The correct answer to the question is **D**. All of the circuits have expressions that are logically equivalent to the original expression.