There are four (4) fish in a large fish bowl. A net is cast to try to catch some fish. It was found that the probability of catching one and only one fish with the net was \( f(1) = 0.26 \). Likewise, it was found that catching two and only two fish was \( f(2) = 0.18 \). Similarly, catching three and only three fish was found to be \( f(3) = 0.12 \) and \( f(4) = 0.06 \).

(a) What is \( f(0) \) the probability of not catching any fish at all?

Ans: \[ f(0) = 1 - f(1) - f(2) - f(3) - f(4) = 1 - 0.26 - 0.18 - 0.12 - 0.06 = 0.38 \]

(b) What is the probability of catching at least two fish?

Ans: \[ f(\text{at least 2 fish}) = f(2) + f(3) + f(4) = 0.18 + 0.12 + 0.06 = 0.36 \]

(c) What is the probability of catching less than two fish?

Ans: \[ f(\text{less than 2 fish}) = f(1) + f(0) = 0.26 + 0.38 = 0.64 \]

(d) What is the probability of catching one fish or more?

Ans: \[ f(\text{1 or more fish}) = 1 - f(0) = 1 - 0.38 = 0.62 \]

Alternatively \[ f(\text{1 or more fish}) = f(1) + f(2) + f(3) + f(4) = 0.26 + 0.18 + 0.12 + 0.06 = 0.62 \]

Question 2

What is the mean of the probability distribution in Question 1?

Ans: \[ \mu = \sum x.f(x) = 0 \times 0.38 + 1 \times 0.26 + 2 \times 0.18 + 3 \times 0.12 + 4 \times 0.06 = 1.22 \]

Question 3

What is the variance of the probability distribution in Question 1?

Ans: \[ \sigma^2 = \sum (x - \mu)^2 . f(x) \]


Thus $\sigma^2 = 1.5316$

Question 4

What is the standard deviation of the probability distribution in Question 1?

Ans: The standard deviation is $\sigma = \sqrt{1.5316} = 1.2376$ to 4 d.p.

Question 5

A probability distribution is given by the function $h(z) = \frac{z^2}{30}$ for $z = 0, 1, 2, 3, 4, 5$. What is the mean and standard deviation of this probability distribution?

Ans: $\mu = \sum z.h(z)$ for $z = 0, 1, 2, 3, 4$

Using a table, we calculate the probability of each value of $z$ using the function $h(z) = \frac{z^2}{30}$

$$
\begin{array}{|c|c|c|c|c|}
\hline
z & h(z) = \frac{z^2}{30} & z.h(z) \\
\hline
0 & 0 & 0 \\
1 & 0.033333 & 0.033333 \\
2 & 0.133333 & 0.266667 \\
3 & 0.3 & 0.9 \\
4 & 0.533333 & 2.133333 \\
\hline
\sum & & 3.333333 \\
\hline
\end{array}
$$

The mean is 3.33 to 2 d.p.

The variance is given by $\sigma^2 = \sum (z - \mu)^2.h(z)$

Again, using a table as follows:
The variance is 0.688889 to 6 d.p.

Now standard deviation $\sigma = \sqrt{0.688889} = 0.83$ to 2 d.p.

Question 6

The probabilities of getting 0, 1, 2, 3, or 4 heads in four flips of a coin are, respectively, $\frac{1}{16}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{4}$, and $\frac{1}{16}$. A person might think that that if you flip a coin four times that 50% of the time you would get two heads and two tails. However, this only occurs $\frac{3}{8}$ or 37.5% of the time. Explain why this is the case.

Also, calculate the mean and standard deviation of the probability distribution and explain what the mean and standard deviation mean in respect to the distribution.

Ans: When we flip a coin four times, there are 16 equally likely outcomes as follows: HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, HTTH, THHT, THTH, TTHH, HTTT, THTT, TTHT, TTTH, and TTTT. We can see that the probability of no heads (ie. TTTT) is $\frac{1}{16}$. Likewise, the probability of all heads (ie. HHHH) is also $\frac{1}{16}$. If we now count all the times that we find exactly two heads and two tails, we find a total of six giving a probability of $\frac{6}{16} = \frac{3}{8}$ or 37.5%. If we rejected all outcomes where all four flips were only heads or only tails, then we would see two heads and two tails appearing 50% of the time. But it is not the case here when we consider all possible outcomes.

The mean is given by $\mu = \sum x . f(x) = 0 . \frac{1}{16} + 1 . \frac{1}{4} + 2 . \frac{3}{8} + 3 . \frac{1}{4} + 4 . \frac{1}{16} = 2$

A value of 2 for the mean indicates that the most likely expected value or result of flipping a coin four times is two heads and two tails.
We can calculate the variance by using the formula $\sigma^2 = \sum x^2.f(x) - \mu^2$

Using a table, we have the following:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$x^2$</th>
<th>$f(x)$</th>
<th>$x^2.f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.0625</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>0.375</td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>0.25</td>
<td>2.25</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>0.0625</td>
<td>1</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td></td>
<td></td>
<td>5</td>
</tr>
</tbody>
</table>

$\sigma^2 = \sum x^2.f(x) - \mu^2 = 5 - 4 = 1$

Thus standard deviation is $\sqrt{1} = 1$

The standard deviation doesn’t have a lot of meaning here. We could make a statement that there is about 68% probability we will get between 1 and 3 heads appearing in four flips of a coin. In actual fact, the probability is 87.5% of getting between 1 and 3 heads.