Chapter 14  Solving equations

An equation has a left-hand side and a right-hand side with an “=” sign between each side. This means that whatever is on the left-hand side is equal to the right-hand side. When we first look at an equation, we might not be able to see that the left-hand side equals the right-hand side. This is due to there being an unknown quantity present. Consider the following equation:

\[ 5 + 10 = 20 \]

On first inspection, the left-hand doesn’t look like it is equal to the right-hand side because of the unknown quantity \( x \). However, we may be able to find a value of \( x \), multiply this value by 5, add 10 and get the value of 20. Basically, by selecting a particular value of \( x \), we can make the left-hand side equal the right-hand side. In this example, if we make \( x = 2 \), the left-hand side will equal the right-hand side. We say that \( x = 2 \) is the solution (or alternatively, a root) of the equation because this value of \( x \) satisfies the equation.

The above equation has only one solution, but we will come across cases where there exists more than one solution for different types of equations.

14.1  Solving linear equations

The first type of equation we will look at is a linear equation and called linear because it is of the form \( ax + b = 0 \). The letter \( a \) represents a number and is called the coefficient of the unknown quantity \( x \), while the number \( b \) is called the constant term. The equation \( 5 + 10 = 20 \) is actually a linear equation and while it is not in the format of \( ax + b = 0 \), it can be modified to this form as follows:

Consider \( 5 + 10 = 20 \). Suppose we subtract 20 from both sides. Both sides will still be equal because we are taking exactly the same amount from both sides.

\[ 5 + 10 - 20 = 20 - 20 \]

Now we just do some adding and subtracting and we get \( 5 - 10 = 0 \) and this is now in the format of \( ax + b = 0 \).

**NB. It should be clearly noted that the term \( x \) must be of power 1. As soon as \( x \) has a power other than 1 in an equation, it is no longer a linear equation.**

Now, let’s solve for \( x \) in the equation \( 5 - 10 = 0 \). Firstly, we will add 10 to both sides as follows:

\[ 5 + 10 + 10 = 0 + 10 \]

And after adding and subtracting the numbers in the equation, we get

\[ 5x = 10 \]
Now, we will divide both sides by 5 as follows:

\[
\frac{5x}{5} = \frac{10}{5}
\]

If we now cancel common factors we get the following:

\[x = 2\]

This is the solution. You will notice that we actually isolated \(x\) in the process of finding the solution. By isolating and making \(x\) the subject of the equation, we have found the solution of the equation.

Consider the following more complex equation:

\[5 - 2x = 2 + 3x\]

To solve, we try to isolate \(x\) and make it the subject of the equation. We have terms involving \(x\) on both sides of the equation. We could start by trying to remove the term \(3x\) from the right-hand side of the equation. We know that if we subtract \(3x\) from \(3x\) we will have zero. So, we will start by subtracting \(3x\) from both sides.

\[5 - 2x - 3x = 2 + 3x - 3x\]

Now, we simplify the equation by adding like terms and get

\[5 - 5x = 2\]

Now we will subtract 5 from both sides as follows:

\[5 - 5x - 5 = 2 - 5\]

Again, adding and subtracting like terms we get

\[-5x = -3\]

We now divide both sides by 5 to isolate \(x\) as follows:

\[-\frac{5x}{5} = -\frac{3}{5}\]

Simplifying we get \(x = -\frac{3}{5}\)

We can then finally remove the negative sign by multiplying both sides by -1 as follows:
\((-1)(-x) = (-1)(-\frac{3}{5})\)

We can now finally isolate \(x\) and made it the subject of the equation as follows:

\[ x = \frac{3}{5} \]

And this is the solution of the equation.

### 14.2 Solving simultaneous equations

Sometimes, we have equations with more than one unknown quantity. For example, consider the following:

\[ x + 2y = 14 \]

We can find values for \(x\) and \(y\) to make both sides of the equation equal each other by assuming a value for one term and then trying to find a corresponding value for the other. We can start by letting \(y = 1\). We now have

\[ x + 2(1) = 14 \]

Thus \( x + 2 = 14 \)

Subtracting 2 from both sides, we get \( x = 12 \) which is a value for \(x\) that makes both sides equal.

If we now start with a value of \(y = 2\), we end up with \( x = 10 \).

For \(y = 3\), \(x = 8\). For \(y = 4\), \(x = 6\). For \(y = 5\), \(x = 4\).

You can see by now that we can go on for ever finding values for \(x\) and \(y\) that can make both sides of the equation equal, but we haven’t found a unique solution. We can say that we have found many possible solutions, but we don’t know which one is the unique solution.

Let’s consider a second equation and find possible solutions as follows:

\[ 3x + y = 17 \]

For \(y = 1\), \(x = \frac{16}{3}\). For \(y = 2\), \(x = \frac{15}{3} = 5\). For \(y = 3\), \(x = \frac{14}{3}\). For \(y = 4\), \(x = \frac{13}{3}\).

Finally, for \(y = 5\), \(x = \frac{12}{3} = 4\). We can go on for ever, but we will stop here.
Again, with the second equation, there are numerous possible solutions. But, what if we try to find a solution for one equation that is also a solution for the second one. By simply looking through the possible solutions for both equations, we find a set of values for $x$ and $y$ that satisfies both equations at the same time or simultaneously. This occurs when $x = 4$ and $y = 5$. We have found a unique solution for both equations simultaneously. In mathematical terms, we say that we have solved the simultaneous equations.

Instead of trying to solve simultaneous equations like we did above, we can take an alternate approach. We can take one equation and isolate one of the unknowns in terms on the second unknown.

Consider the same two equations:

\[
\begin{align*}
x + 2y & = 14 \\
3x + y & = 17
\end{align*}
\]

Let’s isolate $y$ in the second equation and make it the subject. We can achieve this by subtracting $3x$ from both sides as follows:

\[
3x + y - 3x = 17 - 3x
\]

The resulting equation is $y = 17 - 3x$

Using this value of $y = 17 - 3x$, we can substitute it into the first equation as follows:

\[
x + 2(17 - 3x) = 14
\]

Basically, where $y$ was in the first equation we have replaced with (or substituted by) $17 - 3x$.

Removing the brackets, we get $x + 34 - 6x = 14$

We now add like terms and get $-5x + 34 = 14$

We now isolate $x$ and make $x$ the subject. Firstly, we subtract 34 from both sides as follows:

\[
-5x + 34 - 34 = 14 - 34
\]

This gives us $-5x = -20$. Finally, we divide both sides by -5.

\[
\frac{-5x}{-5} = \frac{-20}{-5}
\]

which gives us the solution of $x = 4$.

To find the matching solution for $y$ we can substitute our solution for $x$ into either equation.
Using the first equation we get \( 4 + 2y = 14 \). When we isolate \( y \) and make it the subject, we get \( y = 5 \).

If we had used the second equation \( 3x + y = 17 \) and substituted for \( x = 4 \) into this equation, we would have \( 3(4) + y = 17 \) giving again \( y = 5 \) as the solution.

### 14.3 Solving simultaneous equations

A quadratic equation is of the form \( ax^2 + bx + c = 0 \) where \( a \) is the coefficient of the \( x^2 \) term, \( b \) the coefficient of the \( x \) term and \( c \) a constant term.

Again, \( x \) is the unknown we are trying to solve. One means of solving is to try to factorise the left-hand side.

Consider the quadratic \( x^2 + 3x - 10 = 0 \). Using the techniques we learnt from last week’s lecture, we can factorise the equation as follows:

\[
x^2 + 3x - 10 = (x + 5)(x - 2) = 0
\]

Now, if \((x + 5) = 0\), we have \((0)(x - 2) = 0\). The left-side is then equal to the right side. From this we can deduce that a solution exists when \((x + 5) = 0\). By subtracting 5 from both sides to isolate \( x \), we get \( x = -5 \) as a solution. Likewise, the second solution will occur when \((x - 2) = 0\) giving the solution \( x = 2 \). A quadratic equation will have two solutions for the unknown which in this case is \( x \).

In some cases, only one solution exists. Consider the equation \( x^2 + 4x + 4 = 0 \). The solution is \( x = -2 \) because when we factorise, each factor is repeated.

\[
x^2 + 4x + 4 = (x + 2)(x + 2) = 0
\]

There is also a formula for solving quadratic equations.

If \( ax^2 + bx + c = 0 \), then \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

Using the equation \( x^2 + 3x - 10 = 0 \) and applying the formula, we have the following:

\[
a = 1, b = 3 \text{ and } c = -10
\]

The solutions for \( x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-10)}}{2(1)} = \frac{-3 \pm \sqrt{9 + 40}}{2} = \frac{-3 \pm \sqrt{49}}{2} = \frac{-3 \pm 7}{2}
\]

One solution is \( x = \frac{-3 + 7}{2} = \frac{4}{2} = 2 \).
The other solution is \( x = \frac{-3 - 7}{2} = \frac{-10}{2} = -5 \)

We can see that the solution obtained by using the formula is the same as the factorising method.

**NB.** The formula takes the square root of the quantity \( b^2 - 4ac \). If \( b^2 - 4ac \) is positive in value, there is no problem finding the square root. The two solutions found in this case are called distinct real roots. In certain cases \( b^2 - 4ac = 0 \). In these cases there is only one distinct solution or single root (sometimes called a repeated root). If \( b^2 - 4ac < 0 \), then there are no real roots as we cannot find the square root of a negative value using normal mathematics that would yield a real value.

**Chapter 15**    Sequences and series

**15.1**    Sequences

A sequence is a set of numbers written down in a specific order. The order doesn’t have to be ascending or descending, nor does any particular term in the sequence needs to be related in any way to any other term. However, a sequence can be in a particular order and can be related to other terms present.

Sometimes the symbol “…” is used to indicate that the sequence continues in like manner to the sequence leading up to this symbol. For example, we can have a sequence like 1, 2, 3 … 19, 20. The “…” indicates that all the numbers between 3 and 19 are included in this sequence. It is just a shortcut way of writing a sequence without having to write all the terms in the sequence.

A sequence where the first term and last term are clearly indicated is called a finite sequence. However, some sequences go on forever. 1, 3, 5, 7, 9 … indicates an infinite sequence. The use of “…” without writing a final term indicates an infinite sequence.

Often in sequences, subscript notation is used to denote the sequence. For example, suppose we want to denote the sequence 1, 3, 5, 7, 9 by \( x \). We can let the first term of the sequence be \( x_1 \), the second one \( x_2 \) and so on. We can write the sequence as \( x_1, x_2, \ldots, x_n \).

**NB.** Sometimes, the first term is denoted \( x_0 \), the second \( x_1 \) and so on.

The terms of a sequence can often be found using a formula. For example, we could state that the terms of a sequence \( x \) can be given by \( x_k = 2k + 3 \). A limit can be placed on the number of terms in the sequence, or it can be left as an infinite sequence with no limit on the number of terms. If we want the first three terms of the sequence, we
simply calculate \(x_1, x_2, \text{and } x_3\). To calculate \(x_1\), we simply substitute \(k = 1\) into the formula as follows:

\[ x_1 = 2(1) + 3 = 2 + 3 = 5 \]

Likewise, \(x_2 = 2(2) + 3 = 4 + 3 = 7\) and \(x_3 = 2(3) + 3 = 6 + 3 = 9\)

### 15.2 Arithmetic progressions

There are some special sequences where each term is related to the value of the previous term. For example, if we know the first term, then for each subsequent term we add exactly the same value, we get a special sequence called an arithmetic progression (or arithmetic sequence). There is a common and constant difference between any two terms in the arithmetic progression.

Consider the sequence of 1, 7, 13, 19 … We can see that there is a common difference of 6 between each term. As a result, this sequence is an arithmetic progression.

An arithmetic progression can be expressed in the following manner:

\[ a, a + d, a + 2d, a + 3d, \ldots \]

where \(a\) is the first term and \(d\) is the common difference.

**NB.** The \(n\)th term of an arithmetic progression is given by \(a + (n - 1)d\).

Suppose we want to find the 16\(\text{th}\) term of the arithmetic progression with the first term 2 and common difference 5.

Using the above formula for the \(n\)th term we have

\[ 16\text{th term} = 2 + (16 - 1)5 = 2 + 15(5) = 77 \]

### 15.3 Geometric progressions

A geometric progression (or sequence) is very similar to an arithmetic progression, but instead of adding a common difference to one term to get the next, a common multiple (called a common ratio) is used to calculate the next term.

2, 10, 50, 250 … is an example of a geometric progression where 2 is the first term and 5 is the common ratio. The 2\(\text{nd}\) term is simply \(2 \times 5 = 10\). The 3\(\text{rd}\) term is simply \(10 \times 5 = 50\) and so on.

A geometric progression can be written as \(a, ar, ar^2, ar^3, \ldots\) with \(a\) being the first term and \(r\) the common ratio.
NB. The nth term of a geometric progression is given by \( ar^{n-1} \).

Suppose we want to find the 7th term of the geometric progression that has first term 2 and common ratio 3. Using the formula for the nth term we have

\[
7\text{th term} = 2(3^{7-1}) = 2(3^6) = 2(729) = 1458
\]

### 15.4 Infinite sequences

An infinite series as mentioned earlier goes on for ever. We denote this by using “…” at the end of a sequence.

Consider the sequence \( x_k = \frac{1}{k}, k = 1, 2, 3, 4, 5... \) which represents the sequence

\[
1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}...
\]

As \( k \) gets larger and larger, the subsequent terms get smaller and smaller. As \( k \) tends to infinity, \( \frac{1}{k} \) tends to zero. We say that the limit of this sequence is zero and expressed by the following:

\[
\lim_{k \to \infty} \frac{1}{k} = 0
\]

When a sequence processes a limit, it is said to converge. Alternately, if we have a sequence where each successive term gets larger in value, the sequence is said to diverge.

Consider \( \lim_{k \to \infty} (3 + \frac{1}{k^2}) \). The term \( \frac{1}{k^2} \) tends to zero as \( k \) tends to infinity. We are still left with the value of 3. So the limit of the sequence is 3.

### 15.5 Series and sigma notation

If the terms of a sequence are added together, the result is known as a series. A series is simply the sum of a sequence.

It is not too difficult to add together all the terms of a finite sequence, but when we attempt to calculate the sum of an infinite sequence, it can be difficult. If we can find a finite sum of an infinite sequence, we say that the sequence is converging. Otherwise, if the sequence is diverging, we will never find the sum as the sum will be getting larger and larger as the terms get larger and larger.

The sigma notation \( \sum \) provides a concise and convenient way of expressing long sums. The sum of the sequence 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 can be written as follows:
Consider the sequence 3, 6, 12, 24, 48, 96, how would we express the sum?

The first term is 3. There is a common ratio of 2. There are 6 terms in total. We could express the series (or sum) as follows:

\[ \sum_{k=1}^{6} 3(2^{k-1}) = 189 \]

15.6 Arithmetic series

If the terms of an arithmetic sequence are added together, the result is known as an arithmetic series.

Being more specific, let’s consider an arithmetic progression with the first term 4 and common difference 5. The first five terms of the sequence or progression is 4, 9, 14, 19, 24. The arithmetic series is 4+9+14+19+24. The sum of this arithmetic series is 70.

The sum of the first \( n \) terms of an arithmetic series with the first term \( a \) and the common difference \( d \) is denoted by \( S_n \) and given by

\[ S_n = \frac{n}{2} (2a + (n-1)d) \]

What is the sum of the first 10 terms of the arithmetic series with the first term 3 and common difference 4? Using the formula

\[ S_n = \frac{n}{2} (2a + (n-1)d) = \frac{10}{2} (2(3) + (10-1)4) = \frac{10}{2} (6 + (9)(4)) = 210 \]

15.7 Geometric series

If the terms of a geometric sequence are added together, the result is known as a geometric series.

Being more specific, let’s consider a geometric progression with the first term 2 and common ratio 3. The first five terms of the sequence or progression is 2, 6, 18, 54, 162. The arithmetic series is 2+6+18+54+162. The sum of this geometric series is 242.
The sum of the first $n$ terms of a geometric series with the first term $a$ and the common ratio $d$ is denoted by $S_n$ and given by

$$S_n = \frac{a(1 - r^n)}{1 - r} \quad \text{provided} \quad r \neq 1.$$

Find the sum of the first 5 terms with the first term 2 and common ratio 3. Using the formula we have

$$S_5 = \frac{2(1 - 3^5)}{1 - 3} = \frac{2(-242)}{-2} = 242$$

15.8 **Infinite geometric series**

If the terms of an infinite sequence are added together, the result is known as an infinite series. Finding the sum of an infinite series may not always be possible, but is in certain cases.

Consider the special case of an infinite geometric series for which the common ratio $r$ lies between -1 and 1. In such as case, the sum always exists.

The sum of an infinite number of terms of a geometric series with the first term being $a$ and common ratio $r$ is denoted by $S_\infty$ and given by

$$S_\infty = \frac{a}{1 - r} \quad \text{provided} \quad -1 < r < 1$$

Find the sum of the infinite geometric series with the first term 2 and common ratio $\frac{1}{3}$.

Using the formula we have

$$S_\infty = \frac{2}{1 - \frac{1}{3}} = \frac{2}{\frac{2}{3}} = 3$$
Chapter 16 Functions

16.1 Definition of a function

A function is a rule that receives an input and produces an output.

More importantly, a function is a rule that produces a single output for any given input.

16.2 Notation used for functions

A letter is used to represent a function. Commonly, the letter “f” is used to represent a function, but any letter could be used. An example of a simple function is shown below:

\[ f(x) = x + 2 \]

The above expression means that we have a function \( f \) that takes an input (or value assigned to) \( x \) and produces output (or value) \( x + 2 \).

For example, suppose we give \( x \) and value of 1, we would write the function expression above by replacing \( x \) by 1 everywhere that \( x \) occurs as follows:

\[ f(1) = 1 + 2 = 3 \]

Similarly, we could assign a value of 2 to \( x \) as follows:

\[ f(2) = 2 + 2 = 4 \]

We can express a function using a different letter as mentioned above. Consider the following function:

\[ h(x) = \frac{x^2}{3} + 1 \]

When we assign an input (or value) of 6 to \( x \), the output is as follows:

\[ h(6) = \frac{6^2}{3} + 1 = \frac{36}{3} + 1 = 12 + 1 = 13 \]

We don’t even have to always use the letter \( x \). Consider the following function:

\[ g(t) = t^2 + 3t + 2 \]

If we let \( t = 4 \) we get an output as follows:
\[ g(4) = 4^2 + 3(4) + 2 = 16 + 12 + 2 = 30 \]

### 16.3 Composite functions

Consider the function \( f(x) = 2x \). If we gave an input of 3 to the function, (let \( x = 3 \)), we would have an output of \( f(3) = 2(3) = 6 \)

We don’t always have to give a numeric value as an input. For example, if we gave the input of \( 3a \) (let \( x = 3a \)), we simply replace \( x \) in the function with \( 3a \) as follows:

\[ f(3a) = 2(3a) = 6a \]

The output is in terms of the input \( a \). But instead of using the letter \( a \), we could have used \( x \) as follows:

\[ f(3x) = 2(3x) = 6x \]

What we have done is simply replaced every occurrence of \( x \) with \( 3x \) in the original function. All we have done is tripled the input.

Now, if we had simply just added 3 to the input \( x \) in the original function, we would get the function \( f(x + 3) = 2(x + 3) = 2x + 6 \)

Now suppose we have two functions as follows:

\[ f(x) = 2x \quad \text{and} \quad g(x) = x + 3 \]

We could combine these two functions as follows:

\[ f(g(x)) = f(x + 3) = 2(x + 3) = 2x + 6 \]

\( f(g(x)) \) is called a **composite function**.

Is \( f(g(x)) = g(f(x)) \)?

Using the functions just above where \( f(x) = 2x \) and \( g(x) = x + 3 \)

\[ g(f(x)) = g(2x) = 2x + 3 \]

As you can see in this case, \( f(g(x)) \) does not equal \( g(f(x)) \).

### 16.4 Inverse of a function

So far we have considered functions with some input \( x \) producing some output \( y \) for example. Can we find a function that does the reverse process? Can we find a function...
that takes \( y \) (the output of the above function) as input and gives \( x \) (the input of the above function). If such a function exists, we call this the inverse function of the original function. NB. Not all functions have an inverse.

Consider the function \( f(x) = 2x \). This function states that whatever the input value is, the output will be double the input. The inverse function will be one that does the reverse. The input to the reverse function will be halved to give the output.

The inverse function of \( f(x) = 2x \) is \( g(x) = \frac{x}{2} \).

Suppose we want to find the inverse of \( f(x) = 6 - 2x \). We can apply an algebraic approach as follows:

Let \( y = 6 - 2x \). Firstly, we isolate \( x \) and make \( x \) the subject of the equation.

Adding \( 2x \) to both sides we get

\[ y + 2x = 6 - 2x + 2x = 6 \]

Next, we subtract \( y \) from both sides. \( y + 2x - y = 6 - y \), so \( 2x = 6 - y \)

Now, we divide both sides by 2 giving us

\[ \frac{2x}{2} = \frac{6 - y}{2} \quad \text{or} \quad x = \frac{6 - y}{2} \]

Finally, we interchange \( x \) and \( y \) and get \( y = \frac{6 - x}{2} \) which is the inverse function. We denote the inverse of function \( f \) by \( f^{-1} \). For our example above, if \( f(x) = 6 - 2x \), then \( f^{-1} = \frac{6 - x}{2} \).

**Chapter 17 Graphs of functions**

**17.1 The x-y plane**

Consider Figure 17.1 on page 177 of the textbook for this course. The horizontal line through the center of the figure is called the \( x \) axis while the vertical line is called the \( y \) axis. You will also notice a scale on each axis indicating a value for \( x \) on the \( x \) axis and \( y \) on the \( y \) axis.

We can refer to any position on the diagram using values of \( x \) and \( y \). Find the point on the diagram with \((1,2)\) adjacent to it. The “1” in \((1,2)\) refers to the \( x \) value. If we draw a line parallel to the \( y \) axis and crossing the \( x \) axis at the \( x \) value of 1, you will notice that the line passes through the point referenced by \((1,2)\). The “2” refers to the \( y \) value.
If we draw a line parallel to the \( x \) axis and crossing the \( y \) axis at the \( y \) value of 2, you will notice that the line passes through the point referenced by \((1,2)\).

Any position or point on the diagram can be referenced by using the \( x \) coordinate and \( y \) coordinate. The full coordinates of a point on the diagram is given by specifying both the \( x \) and \( y \) coordinates in the form \((x,y)\). In the example above, we looked at a point with coordinates \((1,2)\).

The middle point of the diagram is at \((0,0)\) and called the origin. It is the point where the two axes cross each other.

### 17.2 Inequalities and intervals

When we create a figure with \( x \) and \( y \) axes, and we put a scale on both axes, we can’t cover all possible values to infinity in each direction on each axis. In actual fact, a number of times, we really only require a very small set of values on the \( x \) axis to display all resultant points. These parts of the \( x \) axis are known as \textit{intervals}. On Figure 17.1, the interval shown for the \( x \) axis is from approximately -4.3 to about 5.

Recall that \( \mathbb{R} \) is used to denote all numbers including fractions and decimals from minus infinity to plus infinity, we can show that any interval referenced by values of \( x \) can be denoted by \( x \in \mathbb{R} \). We can also make use of the symbols “\(<\)”, “\(>\)”, “\(\leq\)”, and “\(\geq\)” to describe intervals on the \( x \) axis.

There are three types of intervals as follows:

(a) \textit{The closed interval}. An interval that includes its end points is called a \textit{closed interval}. For example, if all the numbers from 1 to 3, including both the numbers 1 and 3, comprise a interval, this is known as a closed interval and denoted by square brackets, \([1,3]\). The interval \([1,3]\) can be expressed as

\[
\{x : x \in \mathbb{R}, 1 \leq x \leq 3\}
\]

(b) \textit{The open interval}. Any interval that does not include its end points is called an \textit{open interval}. For example, if all the numbers from 1 to 3, excluding numbers 1 and 3, comprise a interval, this is known as a open interval and denoted by round brackets, \((1,3)\). The interval \((1,3)\) can be expressed as

\[
\{x : x \in \mathbb{R}, 1 < x < 3\}
\]

We say that \( x \) is \textit{strictly greater} than 1 and \textit{strictly less} than 3.

(c) \textit{The semi-open or semi-closed interval}. An interval may be open at one end and closed at the other. Such an interval is called \textit{semi-open} or \textit{semi-closed}.

Examples can be \((1,3]\) = \(\{x : x \in \mathbb{R}, 1 < x \leq 3\}\) and \([1,3)\) = \(\{x : x \in \mathbb{R}, 1 \leq x < 3\}\)
17.3 Plotting the graph of a function

Suppose we are asked to plot a graph of \( y = 2x - 1 \) for \(-3 \leq x \leq 3\). We plot the function with a closed interval meaning that the end values of -3 and 3 are included in the plot.

See Figure 17.5 on page 181 of the set textbook.

17.4 The domain and range of a function

The domain of a function is the interval of the \( x \) axis used in the plot of the function.

The range is the set of output values of \( y \) due to the input of all domain values.

Basically, the domain is the set of input values to the function and the range is the output.

17.5 Solving equations using graphs

Consider the function \( y = x^2 + x - 3 \) in the interval \([-3;3]\). See the plot in Figure 17.12 on page 188 of the set textbook. The curved plot of the function crosses the \( x \) axis at two places, when \( x = 1.34 \) and \( x = -2.30 \). These are the two solutions of the function.

NB. The solutions are really an approximation depending on the accuracy and resolution of the plot.

17.6 Solving simultaneous equations graphically

Consider the two simultaneous equations as follows:

\[
\begin{align*}
4x - y &= 0 \\
3x + y &= 7
\end{align*}
\]

By isolating and making \( y \) the subject of both equations we get

\[
\begin{align*}
y &= 4x \\
y &= -3x + 7
\end{align*}
\]

We can see a plot of these two functions in Figure 17.15 of page 191 of the set textbook.

You will notice the two functions are straight lines and they intercept (cut each other) at the point (1,4). The values of \( x \) and \( y \) at this point is the solution for the simultaneous functions. The solution is \( x = 1 \) and \( y = 4 \).
Sometimes, various functions intercept with each other more than once. Each point of intersection is a solution to the simultaneous functions.

Chapter 18       The straight line

18.1       Straight line graphs

Every straight line graph or plot has an equation of the form \( y = mx + c \) where \( m \) and \( c \) are constants.

In the equation \( y = mx + c \), the value of \( c \) gives the \( y \) coordinate of the point where the line cuts the vertical (\( y \)) axis.

In the equation \( y = mx + c \), the value \( m \) is known as the gradient and is a measure of the steepness of the line.

18.2       Finding the equation of a straight line from its graph

We know that the equation will be in the form of \( y = mx + c \) and that \( c \) is the intersection of straight line and the vertical axis. \( c \) is very easy to find from the graph.

If we know the coordinates on two separate points on the straight line, we can calculate the gradient \( m \).

\[
\text{Gradient} = \frac{\text{difference between the } y \text{ coordinates}}{\text{difference between the } x \text{ coordinates}}
\]

Consider Figure 18.4 on page 200 of the set textbook. We have a straight line passing through the points A (2,5) and B (0.5,2) and intercepting the vertical axis at a value of 1. From this we know that \( c = 1 \). We can also calculate \( m \) the gradient.

\[
m = \frac{5 - 2}{2 - 0.5} = \frac{3}{1.5} = 2
\]

We can now write the equation as \( y = 2x + 1 \)

18.3       Gradients of tangents to curves

Consider the equation \( y = x^2 \). This equation has been plotted in Figure 18.7 on page 205 of the set textbook.
The gradient of a curve at any point is equal to the gradient of the tangent at that point. If we consider point (2,4) and draw a straight line that just touches the curve at this point and then calculate the gradient of this straight line, we then have found the gradient of the curve at the particular point.

If we choose another point on the curve, the gradient will be different. The gradient is constantly changing as we move along the curve.