Computing Systems - COIT11224
Assignment 2 - Solutions

Question 1, Solution
Let \( z = e^x \), then we have
\[ e^{2x} = e^x \cdot e^x = z \cdot z = z^2 \]
to write the equation as
\[ z^2 - 9z + 14 = 0. \]
The solutions are
\[ z_1 = 2 \quad \text{and} \quad z_2 = 7. \]
Thus
\[ e^{x_1} = 2 \quad \text{and} \quad e^{x_2} = 7 \]
and finally
\[ x_1 = \ln 2 = 0.6931, \]
\[ x_2 = \ln 7 = 1.9459. \]
Question 2, Solution

We can obtain

\[
\log(x-1) + \log(x+1) = \\
= \log((x-1)(x+1)) = \log(x^2-1).
\]

Thus

\[
\log(x^2-1) = 2 = \log 10^2
\]

and

\[
x^2 - 1 = 10^2 = 100.
\]

We can finally find

\[
x^2 = 101 \quad \text{and} \quad \\
x_1 = +\sqrt{101}, \quad x_2 = -\sqrt{101}.
\]
Question 3, Solution

We can find

\[ A \cdot B = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 4 & 0 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 2 & 4 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 4 \end{pmatrix} = \]

\[ \begin{pmatrix} 1 \cdot 2 + 2 \cdot 0 + 1 \cdot 1 & 1 \cdot 4 + 2 \cdot 3 + 1 \cdot 0 & 1 \cdot 1 + 2 \cdot 1 + 1 \cdot 4 \\ 3 \cdot 2 + 4 \cdot 0 + 0 \cdot 0 & 3 \cdot 4 + 4 \cdot 3 + 0 \cdot 0 & 3 \cdot 1 + 4 \cdot 1 + 0 \cdot 4 \\ 0 \cdot 2 + 1 \cdot 0 + (-1) \cdot 0 & 0 \cdot 4 + 1 \cdot 3 + (-1) \cdot 0 & 0 \cdot 1 + 1 \cdot 1 + (-1) \cdot 4 \end{pmatrix} = \]

\[ \begin{pmatrix} 2 & 10 & 7 \\ 6 & 24 & 7 \\ 0 & 3 & -3 \end{pmatrix} \]

\[ B \cdot A = \begin{pmatrix} 2 & 4 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ 3 & 4 & 0 \\ 0 & 1 & -1 \end{pmatrix} = \]
\[
\begin{pmatrix}
2.1 + 4.3 + 1.0 & 2.2 + 4.4 + 1.1 & 2.1 + 4.0 + 1.1 \\
0.1 + 3.3 + 1.0 & 0.2 + 3.4 + 1.1 & 0.1 + 3.0 + 1.1 \\
0.1 + 0.3 + 4.0 & 0.2 + 0.4 + 4.1 & 0.1 + 0.0 + 4.1
\end{pmatrix}
\]

Question 4, Solution

We have

\[x + 2y = 15\]
\[x - 2y = -5\]

and can denote

\[AX = B, \text{ where}\]
\[A = \begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \end{pmatrix}, \quad B = \begin{pmatrix} 15 \\ -5 \end{pmatrix}.\]
The solution is

\[ X = A^{-1}B. \]

Since

\[ A^{-1} = \frac{1}{-2-2} \begin{pmatrix} -2 & -2 \\ -1 & 1 \end{pmatrix} = \frac{1}{-4} \begin{pmatrix} -2 & -2 \\ -1 & 1 \end{pmatrix} \]

we get

\[ X = \frac{1}{4} \begin{pmatrix} -2 & -2 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 15 \\ -5 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} -20 \\ -20 \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}. \]

Thus \( x = 5 \), \( y = 5 \).
Question 5, Solution

The frequency histogram and frequency polygon are shown below.
Question 6, Solution

The ogive is shown below

```
<table>
<thead>
<tr>
<th>Height (in inches)</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>49.5</td>
<td>10</td>
</tr>
<tr>
<td>52.5</td>
<td>20</td>
</tr>
<tr>
<td>55.5</td>
<td>30</td>
</tr>
<tr>
<td>58.5</td>
<td>40</td>
</tr>
<tr>
<td>61.5</td>
<td>50</td>
</tr>
<tr>
<td>64.5</td>
<td></td>
</tr>
</tbody>
</table>
```

Question 7, Solution

We can write the data in ascending order 64, 66, 68, 70, 72, 73, 75, 80, 80, 86, 91, 94 to find the mean:

\[
\text{mean} = \frac{1}{12} (64 + 66 + 68 + 70 + 72 + 73 + 75 + 80 + 80 + 86 + 91 + 94) = 76.58
\]

Median:

\[
\text{median} = \frac{73 + 75}{2} = 74 \text{ and mode} = 80.
\]
The stem-and-leaf display is

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

**Question 8, Solution**

We can write

\[
\sum_{i=2}^{5} x_i y_i = x_2 y_2 + x_3 y_3 + x_4 y_4 + x_5 y_5.
\]

**Question 9, Solution**

We can write the data in ascending order

64, 66, 68, 70, 72, 73, 75, 80, 80, 86, 91, 94

and find the range \(94 - 64 = 30\).
Let

\[ x_1 = 64, \ x_2 = 66, \ x_3 = 68, \ x_4 = 70, \ x_5 = 72, \ x_6 = 73, \ x_7 = 75, \ x_8 = 80, \ x_9 = 80, \ x_{10} = 86, \ x_{11} = 91, \ x_{12} = 94. \]

The standard deviation is

\[
\sqrt{\frac{\sum_{i=1}^{12} (x_i - \bar{x})^2}{12}} = 9.76
\]

and the variance is

\[
\frac{\sum_{i=1}^{12} (x_i - \bar{x})^2}{12} = 95.174,
\]

where \( \bar{x} = \frac{1}{12} \sum_{i=1}^{12} x_i = 76.58 \).
Question 10, Solution

- The number of ways the juniors can be chosen for the committee is
  \[ \binom{8}{3} = \frac{8!}{3!(8-3)!} = \frac{8!}{3!5!} = 56. \]

- The number of ways the seniors can be chosen is
  \[ \binom{6}{2} = \frac{6!}{2!(6-2)!} = \frac{6!}{2!4!} = 15. \]

- The number of ways the required committee can be chosen is
  \[ 56 \cdot 15 = 840. \]

Question 11, Solution

- The probability of getting a jack at the first selection is \( \frac{4}{52} \). The probability of
getting a jack at the second selection is \( \frac{3}{51} \).

Thus, the probability of getting two jacks is \( \frac{4}{52} \cdot \frac{3}{51} = \frac{1}{13} \cdot \frac{1}{17} = \frac{1}{221} \).

- The probability of getting a club at the first selection is \( \frac{13}{52} \). The probability of getting a club at the second selection is \( \frac{12}{51} \).

Thus, the probability of getting two clubs is \( \frac{13}{52} \cdot \frac{12}{51} = \frac{1}{17} \).
Question 12, Solution

Since the word "distinction" has 11 letters and there are 3 i's, 2 t's and 2 n's the number of permutations is

\[ \frac{11!}{3!2!2!} = 1,663,200. \]

Question 13, Solution

We have

\[ P(b') = 1 - P(D) = 1 - 0.34 = 0.66 \]

Since C and D are mutually exclusive, we obtain

\[ P(C \cap D) = 0, \]

\[ P(C \cup D) = P(C) + P(D) = 0.61 + 0.34 = 0.95. \]
Question 14, Solution

- The probability of dancing \( D \) and meeting someone new \( M \) is
  \[ P(D \cap M) = 0.4 \cdot 0.6 = 0.24. \]

- The probability that he/she either dances or meets someone new to date is
  \[ P(D \cup M) = P(D) + P(M) - P(D \cap M) = \]
  \[ = 0.4 + 0.3 - 0.24 = 0.46. \]

- The probability that he/she has danced if you know that he/she met someone new to date is
\[
P(D \cap M) = \frac{0.24}{0.3} = 0.8
\]

- c) The probability that he/she has neither danced nor met someone new to date is
  \[1 - P(D \cup M) = 1 - 0.46 = 0.54.\]

- d) No, because
  \[P(D) \cdot P(M) \neq P(D \cap M)\]
  \[0.4 \cdot 0.3 \neq 0.24\]
  \[0.12 \neq 0.24.\]

**Question 15, Solution**

The probability that both of the suppliers will deliver the product in the required
time is
\[ P(X) \cdot P(Y) = 0.4 \cdot 0.6 = 0.24. \]
The probability that the first supplier will deliver the product in the required time while the second one not is
\[ P(X) \cdot P(Y') = P(X)(1-P(Y)) = \]
\[ = 0.4 \cdot (1-0.6) = 0.4 \cdot 0.4 = 0.16. \]
The probability that the second supplier will deliver the product in the required time while the first one not is
\[ P(Y) \cdot P(X') = P(Y)(1-P(X)) = \]
\[ = 0.6 \cdot (1-0.4) = 0.6 \cdot 0.6 = 0.36. \]
All together we have
\[ 0.24 + 0.16 + 0.36 = 0.76. \]