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Exam Cover Page

Term: 2002 Autumn Term
Session: T2 - Autumn Session
Academic Institution: Central Queensland University
Academic Group: Faculty of Info & Comm
Academic Career: Undergraduate
Exam Type: Standard

Candidate Name (BLOCK LETTERS): ____________________________

Candidate Number: _______________________________________

Course: 002783 - Quantitative Methods A
Subject Area: STAT
Catalog Number: 11028
Paper Number: 1
Component: All

Duration: 180 minutes
Perusal Time: 10 minutes
Lecturer: Sue Lindsay
Moderator: Rob McDougall
Open/Closed Book: Open Book (Unrestricted)
Contact Number: 07 4150 7089
Contact Number: 07 4930 9486
Release Exam Paper to Candidate: No

Instructor Authorised/Allowed Materials
Any calculator including programmable and graphics
Any unannotated bilingual translation dictionary

Special Instructions to Candidates:

1. Working must be shown to gain full marks on any questions.
2. There are ten (10) questions on the paper. All questions are of equal value. Complete as many questions as you can. It is expected that eight (8) questions can be completed in the time available. At least seven (7) questions are to be successfully completed for the grade of High Distinction and at least five (5) questions for the grade of Distinction.
3. Answers written on this question paper will not be marked.

Examination Office Supplied Materials

2x Examination Booklets
2x Linear Graph Paper

Central Queensland University considers improper conduct in examinations to be a serious offence. Penalties for cheating are exclusion from the University and cancellation with academic penalty from the course concerned.
Question 1
(a) The value of an antique watch at the time of purchase was $550, while in five years time the watch is expected to be worth $850. Assuming that the value of the watch, given by $V$, is a linear function of time, $t$,

(i) express value as a function of time
(ii) identify the fixed cost and variable cost per year and briefly interpret these.

(b) An appliance company manufactures electric grills for which the variable cost per unit is $15 and the fixed cost is $91,000. What should the selling price be for the company to earn a profit of $80,000 on 10,000 grills?

Question 2
(a) A car sales yard has an incentive plan for its salespeople in addition to their base salary. For each of the first ten cars that a salesperson sells, the commission is $200. The commission for each car sold over the 10 cars will increase by $2 per car. For example the commission on the 12th car sold is $204. How many cars must a salesperson sell in order to earn $15,000 in incentive payments?

(b) Rita’s Appliance Rentals will rent out a TV for $12.95 per week and a video for $8.75 per week. Edward’s Rental Emporium will rent out the same model of TV and video together for a single price of $18.65 per week plus a $20 delivery charge. After how many weeks is it cheaper to rent a TV and video from Edward’s Rental Emporium?

Question 3
A large corporation produces a number of different types of electronic goods, such as mobile phones, laptops and handheld calculators. The monthly cost $C$ to produce handheld calculators is

$$C = 5q + 8$$

where $C$ is in thousands of dollars and $q$ thousand is the number of calculators manufactured in a month. It is also known that the revenue from these is

$$R = -0.125q^2 + 20q$$

where $R$ is also in thousands of dollars. The corporation is limited to a maximum production of 150,000 calculators per month. Illustrate, using both a graph and appropriate equations, how the corporation can operate at a profit. How many calculators should the corporation produce in order to maximise profit?

Question 4
(a) The population of a small Australian mining town was 650 in 1990 and has been shrinking at a rate of 2% per year. The population $t$ years later is approximated by

$$P(t) = 650e^{-0.02t}.$$ Sketch the expected population on a graph for the 20 years after 1990 and forecast the population in the town in the year 2005.

(b) The demand function for a product is $p = 100\left(2^{-\frac{q}{2}}\right)$ where $q$ is the number of units and $p$ is the price of one unit. Solve the equation for $q$, expressing your answer in terms of common logarithms. At what output level will the price be $p=10$?
Question 5
(a) After 5 years an investment of $1000 amounts to $1348.85. At what nominal rate compounded monthly was the investment made?

(b) In two years time, Brian wants to have $3000 available in order to update his home computer. How much should he invest now at 7% compounded quarterly to meet this goal?

(c) One of your work colleagues proposes an investment scheme where a $10,000 investment will guarantee the following cash flows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4000</td>
</tr>
<tr>
<td>3</td>
<td>5000</td>
</tr>
<tr>
<td>4</td>
<td>6000</td>
</tr>
</tbody>
</table>

Assuming a rate of 6% compounded quarterly, is the investment profitable?

Question 6
(a) Helen and Peter are about to retire and intend to use most of their life savings to purchase an annuity that will give them monthly payments of sufficient size to meet their living expenses during retirement. They estimate that they will need $2000 at the end of each month for the next 25 years, beginning in one month. If the rate of interest is 5% compounded monthly, how much must they pay for this annuity?

(b) Salini owns a small cosmetics and toiletries boutique, which she believes is in need of re-decorating. In two years time she plans to totally re-fit the boutique and believes this will cost her $25000. In order to provide the funds for this, she sets up a sinking fund into which she places equal payments at the beginning of each month. If the fund earns 7% compounded monthly, how much should each payment be?

Question 7
Construct an amortization schedule for a loan of $12000 repaid by four payments, one per quarter, with interest at 8% compounded quarterly.

Use the headings:

<table>
<thead>
<tr>
<th>Period</th>
<th>Principal Outstanding</th>
<th>Interest</th>
<th>Payment</th>
<th>Principal Repaid</th>
</tr>
</thead>
</table>
Question 8
(a) If \( \begin{bmatrix} 1 & x \\ -2 & 0 \end{bmatrix} + 2 \begin{bmatrix} -2 & 0 \\ y & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \), what must \( x \) and \( y \) be? Briefly explain why.

(b) Calculate (if possible) \( \begin{bmatrix} 1 & 0 & -4 \\ 2 & -3 & -1 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ -2 & -5 \\ 0 & 1 \end{bmatrix} \).

If this is not possible, briefly explain why.

(c) Use Cramer’s rule (or any matrix method) to solve the following system:
\[
\begin{align*}
2x + 3y + 5z &= 10 \\
3x - 4y + 11z &= 10 \\
5x + 7y - 9z &= 3
\end{align*}
\]

Question 9
A store sells two brands of gas heaters. In order to satisfy distribution requirements, it must sell at least twice as many of type B as type A. The stores supplier can supply no more than 75 gas heaters in total per week.

The store can sell type A heaters for $100 and type B heaters for $80. Determine how many of each type of heater must be sold in order to maximise total revenue.

Question 10
(a) The demand function for a product is
\( p = 95 - 1.5q \)
and the average cost function is
\( \overline{c} = 1.25q + 2.5 + \left( \frac{2.8}{q} \right) \).

Use differentiation to determine the level of output that will maximise profit? At what price does this occur and what is the maximum profit?

(b) Derive equations for marginal cost and marginal revenue for the situation in (a). Evaluate both at \( q=10 \).