Unit Code: 84108  Unit Name: QUANTITATIVE METHODS A

Offer Mode: Internal ☐  External ☐  Both ☑

Duration: THREE (3) Hours  Perusal Time: TEN (10) Minutes

Lecturers: MIKE DRUMM  SUE LINDSAY
Telephone Numbers: 4930 9475  4930 9476

Release Examination to candidate: YES ☑  NO ☐
Questions to be answered on Examination Paper: YES ☑  NO ☐
Open Book: YES ☑  NO ☐
Calculators: Type ☑  Any ☐
Dictionaries: Type ☑  Language dictionary ☐

Materials Supplied by lecturer/Authorised Materials: NONE

Instructions to Candidate:

1. This is an OPEN book examination. Students may bring a ‘clean’ copy of the text Introductory Mathematical Analysis, EF Haeussler and RS Paul, Prentice Hall into the examination. (Students may use Post-It Notes or highlighters in the text for referencing purposes but the text should have no additional material written added to it.)

2. Working must be shown to gain full marks on any question.

3. Answers written on this question paper will not be marked.

4. There are ten questions on the paper. All questions are of equal value. Complete as many questions as you can. It is expected that 8 questions can be completed in the time available. At least 7 questions are to be successfully completed for the grade of High Distinction and at least 5 questions for the grade of Distinction.

Central Queensland University considers improper conduct in examinations to be a serious offence. Penalties for cheating are exclusion from the University and cancellation with academic penalty from the course concerned.
Question 1

(a) A company manufactures souvenir badges for clubs and the tourist industry. One hundred badges of a simple design, consisting of a small badge with up to three colours, can be produced for $75, while two hundred badges can be produced for $125. Assuming that the cost $C$ is a linear function of $q$, the number of badges ordered,
   (i) express cost as a function of order quantity, and
   (ii) determine the fixed cost and variable cost per unit.

(b) The BBQEZ company manufactures a particular model of gas-fired barbeque for $85 each with a fixed cost of $2050. If the company sells the barbeques to retailers for $120, how many barbeques need be sold for the company to realize a profit of $3000?

Question 2

(a) A company buys a particular component from one of two suppliers. For any particular order supplier A charges a fixed cost of $1000 plus $14 per component while supplier B charges a fixed cost of $800 plus $16 per component. For what order quantities should the company order the component from supplier A so as to minimise cost?

(b) A sum of $20000 is invested in two accounts, part in an account earning 5% compounded annually and the remainder in an account earning 5.5% compounded annually. If the total interest at the end of the year is $1060, how much was invested in the account earning 5% compounded annually?

Question 3

A convention centre estimates that if it runs a conference for $q$ attendees that its daily costs $C$, in dollars, will be given by

$$C = 7825 + 50q.$$ 

Also its daily revenue is estimated to be

$$R = 400 + 100q - 0.25q^2$$

where $R$ is again in dollars. Illustrate both graphically and algebraically how the centre can operate at a profit if it can accommodate at most 300 attendees.
Question 4

(a) The projected population \( P \) of a city is given by

\[
P = 80000e^{0.04t},
\]

where \( t \) is the number of years after 1995. Sketch the expected population on a graph for the 20 years after 1995 and predict the population for the year 2015.

(b) The demand equation for a consumer product is \( q = 150 - 1.1p \). Solve the equation for \( p \), expressing your answer in terms of common logarithms. Evaluate \( p \) to two decimal places when \( q = 40 \).

Question 5

(a) A credit union advertises a nominal interest rate of 6% compounded quarterly. What is the effective interest rate?

(b) Suppose that $5000 is deposited in an account that pays 9% compounded monthly. Find the amount in that account (to the nearest cent) 5 years later. What is the compound interest earned during this time?

(c) An account earns 5% interest compounded quarterly. How long will it take for a deposit to double in value?

Question 6

(a) An annuity pays $800 at the end of each quarter for the next 5 years starting in 3 months. How much will the annuity cost if the interest rate is 9% compounded quarterly?

(b) A sinking fund is to be established to replace in ten years time a piece of machinery that is estimated will then cost $20000. Given that it is convenient to put aside eight equal payments at the end of each year beginning in two years time, calculate the payments that need be made. Assume an interest rate of 8% compounded annually.

Question 7

Construct an amortization schedule for a loan of $4000 repaid by three payments, one per year, with interest at 12% compounded annually.

Use the headings:

<table>
<thead>
<tr>
<th>Period</th>
<th>Principal Outstanding</th>
<th>Interest</th>
<th>Payment</th>
<th>Principal Repaid</th>
</tr>
</thead>
</table>
Question 8

(a) Calculate

\[
\begin{pmatrix}
3 & 1 & -1 \\
3 & 2 & 4 \\
1 & 5 & 0
\end{pmatrix}
+ (-2)
\begin{pmatrix}
4 & 2 & 0 \\
1 & 1 & 2 \\
1 & 0 & 4
\end{pmatrix}
\cdot
\begin{pmatrix}
2 & 2 & 2 \\
3 & 1 & 1 \\
1 & -1 & 2
\end{pmatrix}
\]

(b) Use any matrix method to solve the system

\[
\begin{aligned}
3r - 4t - 4 &= 0 \\
2r + 3s + 3 &= 0 \\
2r - 3s + t - 2 &= 0
\end{aligned}
\]

Question 9

RTB blends three basic ingredients A, B, C to produce two brands of potting mix which it then sells to retailers in two kilogram bags. The quantity of each ingredient going into the two mixes together, with the available quantity of each ingredient, is summarised below (all quantities in kilograms).

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic Blend</td>
<td>1.2</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Premier Blend</td>
<td>1.0</td>
<td>0.6</td>
<td>0.4</td>
</tr>
<tr>
<td>Available quantities</td>
<td>300</td>
<td>90</td>
<td>70</td>
</tr>
</tbody>
</table>

If RTB supplies the Basic Brand to its retailer for $3.40 and the Premier Brand to its retailer for $4.50, determine how much of each brand should be produced so as to maximise total income.

Question 10

(a) The demand function for a product is

\[ p = 85 - 1.2q \]

and the cost function is

\[ C = 250 + 12q. \]

At what level of output will profit be maximised? At what price does this occur and what is the maximum profit?

(b) If the average cost function of a product is \( \bar{c} = 1.05q + 2.5 + (2.6/q) \) find the marginal cost when \( q = 30. \)