Workshop Exercises for Week 12

Attempt the following workshop exercises. These are the exercises that are used in the workshop portion of the internal lectures. No solutions are available for these exercises. Answers to selected workshop exercises are included.

1. Find and identify the maximum or minimum values of each of the following functions using differentiation.
   (a) \( y = 2x^2 - x + 10 \)
   (b) \( y = 2x^3 - 15x^2 + 36x + 5 \)

2. A company estimates that the expenditure of \( x \) million dollars in advertising will generate \( 9x - 3x^3 \) million dollars in profit.
   (a) How much should the firm spend on advertising to make the profit as large as possible?
   (b) How much would the maximum profit be?

3. A firm produces and sells a single product. The total revenue \( R \) is described by \( R = 500q - 5q^2 - 50 \) and the total cost equation is \( C = q^3 - 10q^2 + 40q \) where \( q \) is the number of units.
   (a) Find the marginal revenue equation.
   (b) Find the marginal cost equation.
   (c) Find the firm’s profit equation.
   (d) Find the marginal profit equation.
   (e) Evaluate each of (a) to (d) above when \( q = 10 \) items are produced.

4. Given the total cost function \( C = q^2 + 4 \) and the total revenue function \( R = 24q - 4q^2 \) where \( q \) is the number of units, answer the following questions.
   (a) Find the profit function.
   (b) Find the level of output that will maximise profit.
   (c) Show that this is the level of output where marginal revenue equals marginal cost.

5. A manufacturer has determined that, for a certain product, the average cost (in dollars per unit) is given by \( \bar{C} = 2q^2 - 36q + 210 - \frac{200}{q} \), where \( q \) is the number of units and \( 2 \leq q \leq 10 \).
   (a) At what level within the interval \([2, 10]\) should production be fixed in order to minimise total cost?
   (b) What is the minimum total cost?
   (c) If production were required to lie within the interval \([5, 10]\), what value of \( q \) would minimise total cost?

6. For a monopolist’s product, the cost function is \( C = 0.004q^3 + 20q + 500 \), and the demand function is \( p = 450 - 4q \) where \( p \) is the price in dollars and \( q \) is the number of units.
   (a) Find the profit-maximising output.
   (b) At what price does the profit-maximising output occur?
   (c) What is the maximum profit?
Answers
1.  a) minimum at (0.25, 9.875)
   b) maximum at (2, 33) and minimum at (3, 32)
2.  a) $1 million
    b) $6 million
3.  e) \( R'(10) = \$400; \ C'(10) = \$140; \ P(10) = \$4050; \ P'(10) = \$260 \)
4.  b) 2.4 units
5.  a) 2 units
    b) $92
    c) 7 units
6.  a) 50 units
    b) $250
    c) $10,500