Important Note: This is a sample exam only. There are multiple topics in each week of the course that may be tested in the end of term exam and which are not included in this sample paper. Note also that space will be provided for writing answers in the end of term exam paper.

Question 1 (4 Marks)

(a) Perform the operation and simplify: \( \frac{8}{x-2} - \frac{5}{x} \).

(b) Simplify, expressing your answer in terms of positive exponents: \( \left( \frac{q^{-1}r^2s^{-3}}{q^2r^{-4}} \right)^2 \).

Question 2 (4 Marks)

A company manufactures portable radios. Its total monthly production cost \( C \) is given by \( C = 4q + 10 \) where \( C \) is in thousands of dollars and \( q \) is the number of radios manufactured in a month (in thousands). It is also known that the total monthly revenue \( R \) from these is given by \( R = -0.125q^2 + 7q \) where \( R \) is also in thousands of dollars. The company is limited to a maximum production of 28,000 radios per month. Over what production levels will the company operate at a profit?

Question 3 (4 Marks)

A company manufactures a total of 10,000 units of its product at its two plants, called A and B. Available data are given below.

<table>
<thead>
<tr>
<th>Plant A</th>
<th>Plant B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit cost for labour and material</td>
<td>$5.00</td>
</tr>
<tr>
<td>Fixed costs</td>
<td>$30,000</td>
</tr>
</tbody>
</table>

Between the two plants the company has decided to allot no more than $117,000 for total costs. What is the minimum number of units that must be produced at Plant A?
Question 4  (6 Marks)

The daily cost $C$ of producing Lustre Lamps is assumed to be a linear function of $q$, the number of lamps produced. Suppose it costs $180 to produce two lamps and $201 to produce five lamps.

(a) Find $C$ as a linear function of $q$ if it is possible to produce as many as 50 lamps per day.

(b) What are the fixed cost and the variable cost per unit under these assumptions?

(c) Using your answer to (a), determine the cost to produce 4 lamps.

(d) Assuming that the function remains accurate, how many lamps will be produced for a cost of $355?

Question 5  (6 Marks)

(a) The projected population $P$ of a city is given by $P = 100,000e^{0.05t}$ where $t$ is the number of years after 1990.

(i) Sketch the expected population on a graph for the 30 years after 1990.

(ii) Forecast the population in the year 1998.

(b) Use the definition of logarithms to rearrange and solve the following equations:

(i) $\log_3 x = 2$

(ii) $\log_e (3t - 6) = 1$

Question 6  (6 Marks)

(a) The demand function for a particular product is $q = 45 - 3^p$ where $q$ is the number of units of the product and $p$ is the price of one unit in dollars.

(i) Solve for $p$, expressing your answer in terms of common logarithms.

(ii) Using your answer to part (a) (i), when 20 units of the product are demanded, what will the price be?

(b) Use logarithms to solve the following expression for $x$: $43e^{4x+8} = 5$. Round your answer to 6 decimal places.

Turn page for Questions 7, 8, & 9
Question 7  

(a) A bank pays 8% interest compounded quarterly. How long will it take for a deposit to triple in value?

(b) An investment opportunity guarantees the following cash flows: $10,000 at the end of two years, $5500 at the end of three years, and a further $5500 at the end of five years. Suppose the initial investment required is $20,000 and the interest rate is 5% compounding annually.

(i) Find the net present value of the cash flows.

(ii) In one sentence, state whether the investment is profitable and why.

Question 8  

(a) Given an annuity with equal payments each month for six years and an interest rate of 7.75% compounded monthly, determine the monthly payment if the future value of the annuity is $25,000.

(b) Dorothy wishes to purchase an annuity that will pay her son Mark $950 at the beginning of each quarter for the next 6 years, starting today. If the rate of interest is 8% compounded quarterly, how much must Dorothy pay for this annuity?

(c) On the day of their son's birth, Mr and Mrs Planner decided to set aside a sum of money to provide for his college education. They wish to make a single deposit in a bank that pays 9% interest compounded yearly in order to provide a payment of $8500 on each of the son's 18th, 19th, 20th, and 21st birthdays. How much should they deposit?

Question 9  

A loan of $3000 is to be repaid by four payments, one per month, with interest at 12% compounded monthly.

(a) Find the monthly repayments.

(b) Construct an amortisation schedule for this loan. Use the following headings in your table and be sure to show any workings. Adjust the final payment, if necessary. Remember to include a totals row at the end of the table.

<table>
<thead>
<tr>
<th>Period</th>
<th>Principal Outstanding</th>
<th>Interest</th>
<th>Payment</th>
<th>Principal Repaid</th>
</tr>
</thead>
</table>

(c) What is the finance charge on this loan?
Question 10 (8 Marks)

(a) The demand function for a product is \( p = 72 - 0.04q \) and the total cost function is \( C = 500 + 3q \). Use the information to find the following:

(i) Use differentiation to find at what level of output profit will be maximized and show it is the maximum.

(ii) At what price does this occur?

(iii) What is the maximum profit earned?

(b) If \( \bar{c} = 0.03q + 1.2 + \frac{3}{q} \) is an average cost function, find the marginal cost when \( q = 100 \). In one sentence, interpret this value.