1. Given the matrices
\[ a = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix} \text{ and } D = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \]
evaluate each of the following expressions, or indicate that the operation is invalid.

(i) \( a + b \), (ii) \( ab \), (iii) \( ab^T \), (iv) \( Db \), (v) \( DC \). [18 marks]

2. A system of linear equations \( A \mathbf{x} = \mathbf{b} \) is such that
\[ A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}. \]
Use Cramer’s rule to determine the solution. [18 marks]

3. Use cofactors to determine the inverse of the matrix
\[ A = \begin{bmatrix} 3 & 0 & 1 \\ 1 & -3 & 0 \\ 1 & 0 & 2 \end{bmatrix}. \]
Verify your answer by evaluating \( AA^{-1} \). [30 marks]

4. Use Gauss-Seidel iteration to solve the system \( A \mathbf{x} = \mathbf{b} \) where \( A \) is the matrix of A question 3 and \( \mathbf{b}^T = [1, 2, 3] \). Start your iterations with the zero vector and, retaining four decimal places in your working, proceed to the third iteration. [30 marks]

5. A transformation is a vertical shear followed by a horizontal expansion. Find the matrix of the transformation if the shear is such that the point \( (2, 3) \) is transformed to \( (2, 5) \), and the expansion is such that the point \( (2, 1) \) is transformed to \( (3, 1) \). [18 marks]

6. Use de Moivre’s theorem to express \((1 - 2j)^6\) in polar form. [24 marks]

7. Use the identity
\[ e^{j(\theta + \phi)} = e^{j\theta} e^{j\phi} = (\cos \theta + j \sin \theta)(\cos \phi + j \sin \phi) \]
to establish the usual identity for \( \cos(\theta + \phi) \). [18 marks]

8. Find all fourth roots of \( 1 + j \), that is, solve the equation \( z^4 - (1 + j) = 0 \). Give your answers in polar form with arguments taking their principal value. [24 marks]