1. We have
   (i) \(f(g(x)) = f(x + 1) = (x + 1)^2 - 1 = x^2 + 2x + 1 - 1 = x^2 + 2x,
   \)
   (ii) \(g(f(x)) = g(x^2 - 1) = (x^2 - 1) + 1 = x^2.

2. The function \(f(x) = 3x - 1\) defined for \(-1 \leq x \leq 4\) (ie the domain of \(f\)) is graphed in figure 1. Note from the figure that the range of \(f(x)\) is \(-4 \leq x \leq 11\).

   From \(y = 3x - 1\) we have \(x = \frac{1}{3}(y+1)\) so that \(f^{-1}(x) = \frac{1}{3}(x+1)\), the graph of which is shown in figure 2. Note that the domain of \(f^{-1}(x)\) is \(-4 \leq x \leq 11\) and range is \(-1 \leq x \leq 4\).
3. The graph of \( f(x) \) is shown in figure 3. Note that the \( y = f(x) \) being given as \( y = x - 1 \) for \( 0 \leq x \leq 4 \) and the periodicity (with period 4) of \( f(x) \) enables us to graph the function over the extended interval.

![Graph of \( f(x) \) for exercise 3.](image)

4. (i) We have \( y = 0 \) for \( x = 2 \).
(ii) \( y \to \infty \) as \( x \to -1 \) and \( x \to 3 \).

5. Since \( x^3 + 2x^2 - x - 2 = 0 \) has a solution \( x = 1 \), \( x^3 + 2x^2 - x - 2 \) has a factor \( x - 1 \). We therefore write

\[
x^3 + 2x^2 - x - 2 = (x - 1)(ax^2 + bx + c)
\]

and comparing coefficients \( a = 1 \), \( b - a = 2 \), \( c - b = -1 \) and \( -c = -2 \). Solving we have \( a = 1 \), \( b = 3 \), and \( c = 2 \) so that

\[
x^3 + 2x^2 - x - 2 = (x - 1)(x^2 + 3x + 2) = (x - 1)(x + 1)(x + 2).
\]

Thus the equation has solutions \( x = -2, x = -1 \) and \( x = 1 \).

6. Starting with

\[
x + 3y = 5 \quad 2x + 5y = 9
\]

we multiply the first equation by 2:

\[
2x + 6y = 10 \quad 2x + 5y = 9
\]

and subtract the second equation from the first to obtain \( y = 1 \). Substituting this value for \( y \) into the first equation we have \( x = 2 \).
7. We write
\[
\frac{3x - 1}{(x - 1)(x + 2)} = \frac{A}{x - 1} + \frac{B}{x + 2}
\]
\[
= \frac{A(x + 2)}{(x - 1)(x + 2)} + \frac{B(x - 1)}{(x - 1)(x + 2)}
\]
\[
= \frac{(A + B)x + (2A - B)}{(x - 1)(x + 2)}
\]
Comparing coefficients we have \(A + B = 3\) and \(2A - B = -1\) These equations give \(A = \frac{2}{3}\) and \(B = \frac{7}{3}\) and the final result
\[
\frac{3x - 1}{(x - 1)(x + 2)} = \frac{2}{3} \frac{x}{x - 1} + \frac{7}{3} \frac{1}{x + 2}.
\]
8. (i) \((e^x)^{-2}e^x = e^{-2x}e^x = e^{-2x+x} = e^{-x},\)
(ii) \(e^{2x}(e^{-x} + e^{-2x}) - e^x = e^{2x}e^{-x} + e^{2x}e^{-2x} - e^x = e^{2x-x} + e^{2x-2x} - e^x = e^x + e^0 - e^x = 1.\)
9. We write
\[
2e^x - e^{-x} = A \sinh x + B \cosh x
\]
\[
= A \frac{e^x - e^{-x}}{2} + B \frac{e^x + e^{-x}}{2}
\]
\[
= \frac{1}{2}(A + B)e^x + \frac{1}{2}(B - A)e^{-x}
\]
and comparing coefficients \(\frac{1}{2}(A + B) = 2\) and \(\frac{1}{2}(B - A) = -1\). Solving we obtain \(A = 3\) and \(B = 1\). We therefore have
\[
2e^x - e^{-x} = 3 \sinh x + \cosh x.
\]
10. We have \(\log 2x - 1 = 4\) or \(\log 2x = 5\). This is equivalent to \(2x = 10^5\) or \(x = 50000.\)