Test 1

1. Given the functions \( f(x) = x^2 - 1 \) and \( g(x) = x + 1 \) determine each of the compositions
   (i) \( f(g(x)) \),  
   (ii) \( g(f(x)) \).  
   [20 marks]

2. Make a sketch of the function \( f(x) = 3x - 1 \) where \(-1 \leq x \leq 4\).
   Find (ie by algebraic manipulation) and plot the inverse of the function. Take care to take account of the domain and the range of the function and its inverse.  
   [20 marks]

3. The function \( y = f(x) \) is such that \( y = x - 1 \) for \( 0 \leq x \leq 4 \). Sketch the function over the interval \(-4 \leq x \leq 8\) if \( f(x) \) is a periodic function with period 4.  
   [16 marks]

4. Consider the rational function \( y = \frac{x - 2}{(x + 1)(x - 3)} \).
   (i) For what values of \( x \) is \( y = 0 \)?  
   (ii) For what values \( a \) does \( y \to \infty \) as \( x \to a \)?  
   [16 marks]

5. Find all solutions of the cubic polynomial equation \( x^3 + 2x^2 - x - 2 = 0 \) given that the equation has a solution \( x = 1 \).  
   [20 marks]

6. Solve the following system of linear equations by elimination.
   \[ x + 3y = 5 \quad 2x + 5y = 9 \]  
   [16 marks]

7. Express the following proper fraction in terms of partial fractions.
   \[ \frac{3x - 1}{(x - 1)(x + 2)} \]  
   [24 marks]

8. Simplify the following expressions.
   (i) \( (e^x)^{-2} e^x \),  
   (ii) \( e^{2x} (e^{-x} + e^{-2x}) - e^x \).  
   [16 marks]

9. Express the function \( f(x) = 2e^x - e^{-x} \) in terms of hyperbolic sine and cosine functions.  
   [20 marks]

10. Solve the equation \( \log 2x - 1 = 4 \) for \( x \).  
    [12 marks]