1. The determinant, expanding by the first row, is
\[
\begin{vmatrix}
1 & 0 & 1 \\
1 & -1 & 2 \\
-1 & 1 & 1
\end{vmatrix}
= 1 \times \begin{vmatrix}
-1 & 2 \\
1 & 1
\end{vmatrix}
- 0 \times \begin{vmatrix}
1 & 2 \\
-1 & 1
\end{vmatrix}
+ 1 \times \begin{vmatrix}
1 & 2 \\
1 & 1
\end{vmatrix}
= 1 \times (-3) - 0 \times (3) + 1(0) = -3.
\]

2. The minors are
\[M_{11} = 4,\ M_{12} = 3,\ M_{21} = 2,\ M_{22} = 1\]
and the cofactors
\[A_{11} = 4,\ A_{12} = -3,\ A_{21} = -2,\ A_{22} = 1.\]
Also \(|A| = -2.\)
We then have
\[
A^{-1} = \frac{1}{2} \begin{bmatrix}
4 & -3 \\
-2 & 1
\end{bmatrix}^T = \begin{bmatrix}
-2 & 1 \\
\frac{3}{2} & -\frac{1}{2}
\end{bmatrix}.
\]
The answer is confirmed from
\[
AA^{-1} = \begin{bmatrix}
1 & 2 \\
3 & 4
\end{bmatrix} \begin{bmatrix}
-2 & 1 \\
\frac{3}{2} & \frac{1}{2}
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]

3. (i) We compute
\[
x = A^{-1}b = \begin{bmatrix}
-2 & 1 \\
\frac{3}{2} & \frac{1}{2}
\end{bmatrix} \begin{bmatrix}
1 \\
2
\end{bmatrix} = \begin{bmatrix}
0 \\
\frac{1}{2}
\end{bmatrix}
\]
(ii) The augmented matrix of the system is
\[
\begin{bmatrix}
1 & 2 & 1 \\
3 & 4 & 2
\end{bmatrix}
\]
and subtracting three times row 1 from row 2 we have
\[
\begin{bmatrix}
1 & 2 & 1 \\
0 & -2 & -1
\end{bmatrix}.
\]
The solution \(x_1 = 0,\ x_2 = \frac{1}{2}\) may be obtained by either solving the equations here in reverse order or alternatively by applying further row operations (add row 2 to row 1 and divide row 2 by \(-2\)) to obtain
\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & \frac{1}{2}
\end{bmatrix}.
4. Rearranging the equations in the usual way we have

\[
\begin{align*}
    x_1^{(i+1)} &= 1 - \frac{1}{3}x_2^{(i)} - \frac{1}{3}x_3^{(i)} \\
    x_2^{(i+1)} &= \frac{4}{3} + \frac{1}{3}x_3^{(i)} \\
    x_3^{(i+1)} &= -\frac{1}{2} - \frac{1}{2}x_1^{(i+1)}
\end{align*}
\]

The calculation is set out below.

\[
\begin{array}{c|cccc|c}
  i & 0 & 1 & 2 & 3 & \text{Exact} \\
  x_1^{(i)} & 0.0000 & 1.0000 & 0.8889 & 0.9815 & 1 \\
  x_2^{(i)} & 0.0000 & 1.3333 & 1.0000 & 1.0185 & 1 \\
  x_3^{(i)} & 0.0000 & -1.0000 & -0.9444 & -0.9907 & -1
\end{array}
\]

5. The characteristic equation is \( \lambda^2 = 4\lambda - 3 = 0 \) or \( \lambda^2 - 4\lambda + 3 = (\lambda - 1)(\lambda - 3) = 0 \). Thus \( \lambda = 1 \) and \( \lambda = 3 \) and the complementary solution is \( (t_n)_c = c_1(1)^n + c_2(3)^n = c_1 + c_23^n \). To obtain the particular solution we trial \( t_n = An \) and substitution gives

\[ An = 4A(n - 1) - 3A(n - 2) + 2. \]

Simplifying we obtain \( A = -1 \). The particular solution is then \( (t_n)_p = -n \) and the general solution is thus \( t_n = (t_n)_c + (t_n)_p = c_1 + c_23^n - n \).

6. First we note that the probability that a coach will be more than five minutes late is 0.05 and that it will be more than ten minutes late is 0.03. Thus the probability that a coach that is more than five minutes late will be more than ten minutes late is \( \frac{0.03}{0.05} = 0.6 \).

7. (i) \( P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.6 + 0.7 - 0.4 = 0.9 \)
(ii) \( P(A|B) = P(A \cap B)/P(B) = 0.4/0.7 \approx 0.5714 \).

8. The calculation is set out below.

\[
\begin{array}{c|cccc|c}
  n & 0 & 1 & 2 & 3 & 4 & \Sigma \\
  P(n) & 0.84 & 0.06 & 0.06 & 0.03 & 0.01 & \\
  nP(n) & 0.00 & 0.06 & 0.12 & 0.09 & 0.04 & 0.31 \\
  n^2P(n) & 0.00 & 0.06 & 0.24 & 0.27 & 0.16 & 0.73
\end{array}
\]

We thus have mean \( E(N) = \Sigma nP(n) = 0.31 \), and variance \( \text{Var}(N) = \Sigma n^2P(n) - (\Sigma nP(n))^2 = 0.73 - (0.31)^2 = 0.6339 \). The standard deviation is then 0.7962.