Applications of trigonometry

We here consider further applications of trigonometry.

Example Consider the situation shown in figure 1 in which we observe a lighthouse from out to sea. Knowing the light house to be 55m high we measure the angle of elevation of the base of the light house to be 10° and the angle of elevation of the top of the light house to be 13°. From this information determine the height of the light house above sea level and the distance of the observer from the light house (that is the distances $x$ and $y$ in figure 1).

Figure 1: Notation for example 1.

From the right angle triangles of figure 1 we have

\[ x = y \tan 10^\circ \quad \text{and} \quad x + 55 = y \tan 13^\circ. \]

Eliminating $x$ between these two equations we have

\[ y \tan 10^\circ + 55 = y \tan 13^\circ \]

or

\[ y = \frac{55}{\tan 13^\circ - \tan 10^\circ}. \]

Thus the distance of the observer from the light house is
\[ y = \frac{55}{0.2309 - 0.1763} = \frac{55}{0.0545} = 1,008 \text{ metres.} \]

For the height of the light house above sea level we have

\[ x = y \tan 10^\circ = 1008 \times 0.1763 = 177.8 \text{ metres.} \]

**Surveying**

We note that in surveying applications it is usual to measure angles in degrees with 60 minutes to the degree and sixty seconds to the minute. Thus 0.7678 radians is 0.7678 \times (180/\pi) = 43.99 degrees or 43 degrees, 59 minutes and 30 seconds.

**Example** Consider the problem of determining the height \( h \) of the tower in figure 2. The tower is situated on a flat surface and, as a means of determining the height of the tower, we take three measurements of the angle of elevation of the top of the tower along the straight line \( ABC \) in figure 2. The distances \( x \) and \( y \) are known.

![Figure 2: Notation for the surveying problem.](image)

From the right angle triangle \( ADE \) (see figure 3) we have \( AD = h \cot \alpha \). Similarly \( BD = h \cot \beta \) and \( CD = h \cot \gamma \).

From the ground plan as shown in figure 4 we have, applying the cosine rule in turn to triangles \( ABD \) and \( ACD \)

\[ h^2 \cot^2 \beta = h^2 \cot^2 \alpha + x^2 - 2x(h \cot \alpha) \cos \phi \]
\[ h^2 \cot^2 \gamma = h^2 \cot^2 \alpha + (x + y)^2 - 2(x + y)(h \cot \alpha) \cos \phi. \]

Solving these two equations for \( \phi \) and equating the two expressions we have
\[
\frac{h^2 \cot^2 \alpha + x^2 - h^2 \cot^2 \beta}{2x(h \cot \alpha)} = \frac{h^2 \cot^2 \alpha + (x + y)^2 - h^2 \cot^2 \gamma}{2(x + y)(h \cot \alpha)}.\]

We can now solve for \( h \) to obtain
\[
h = \sqrt{\frac{xy(x + y)}{x(\cot^2 \gamma - \cot^2 \beta) + y(\cot^2 \alpha - \cot^2 \beta)}}.\]

**Bearings**

A *bearing* is an angle a direction in the horizontal plane makes with due north, usually measured in degrees and measured positive in a clockwise direction.

**Example** A trawler travels at a bearing of 45° (that is NE) at 24 knots for 4 hours and then at 157.5° (SSE) at 18 knots for 6 hours. On what bearing is
the starting point from the final point and how long would it take to return to the starting point travelling at 20 knots.

Note that a knot is one nautical mile per hour and one nautical mile is 1852 metres approximately.