Further trigonometry

We consider the problem of ‘solving’ triangles, that is the problem of finding the lengths of the sides and the angles of a triangle from given information. The trigonometric functions are particularly useful here.

Right angle triangles

To ‘solve’ right angle triangles we will usually be given two quantities for the triangle (an angle and a side say) and are required to solve for the remaining quantities (sides and angles). We adopt the notation of figure 1 ($\angle CAB$ is $A$, length of side opposite the angle $A$ is $a$ etc).

\[
\begin{align*}
\text{Figure 1: Notation for the right angle triangle.}
\end{align*}
\]

For right angle triangles we have recourse to Pythagoras’s theorem

\[a^2 + b^2 = c^2\]

since this allows us, given the lengths of two sides, to determine the length of the third. Also (for the general triangle) we may use the fact that the three angles of a triangle sum to $180^\circ$ so that, given two angles, we can readily determine the third.

Next, given an angle $\theta$ and the length of a side (specified to be either adjacent, opposite or hypotenuse relative to $\theta$) we can use one of the trigonometric ratios

\[\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}\]
\[
\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}
\]

\[
\tan \theta = \frac{\text{opposite}}{\text{adjacent}}
\]

to determine the other sides.

**Example**

**The sine rule**

For triangles that do not include a right angle we have the *sine rule* (refer to the notation of figure 2)

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

![Figure 2: Notation for the sine rule.](image-url)
The sine rule applies when either (a) one side and two angles or (b) two sides and a non included angle are given.

Example

The cosine rule

The cosine rule is

\[ a^2 = b^2 + c^2 - 2bc \cos A \]

(figure 3) or similar equations involving \( \cos B \) and \( \cos C \).

The cosine rule applies when either (a) three sides or (b) two sides and the included angle are specified.

![Figure 3: Notation for the cosine rule.](image)

Example