1. (i) \((3 + 4j) - (2 - 6j) = (3 - 2) + (4 + 6)j = 1 + 10j.\)
   (ii) \((3 + 4j)(2 - 6j) = 6 - 18j + 8j - 24j^2 = 30 - 10j.\)
   (iii) \(\frac{3 + 4j}{2 - 6j} = \frac{3 + 4j}{2 - 6j} \frac{2 + 6j}{2 + 6j} = \frac{-18 + 26j}{40} = -\frac{9}{20} + \frac{13}{20}j.\)

2. (i) \(x^2 + 2x + 5 = (x + 1)^2 + 4 = 0\) gives \((x + 1)^2 = -4\) so that \(x + 1 = \pm 2j\) or \(x = -1 \pm 2j.\)
   (ii) \(z^2 - 3z + 7 = (x - \frac{3}{2})^2 + \frac{19}{4} = 0\) or \((x - \frac{3}{2})^2 = -\frac{19}{4}.\) Thus \(x = \frac{3}{2} \pm \frac{\sqrt{19}}{2}j.\)

3. (i) \(3 + 4j = 5\arctan \frac{4}{3} = 5\arctan 0.927.\)
   (ii) \(2 - 6j = \sqrt{40\angle - \arctan 3} = \sqrt{40\angle (-1.249)}\)
   (iii) \((3 + 4j)(2 - 6j) = 5\sqrt{40\angle ((0.927 + (-1.249)))} = 5\sqrt{40\angle (-0.321)}.\)
   (iv) \(\frac{3 + 4j}{2 - 6j} = \frac{5}{\sqrt{40\angle ((0.927 - (-1.249)))}} = \frac{5}{\sqrt{40\angle 2.176}.\)

4. If \(z = x + jy\) then \(z = 3z + 4j\) gives \(x + jy = 3(x - jy) + 4j\) or, equating real and imaginary parts, \(x = 3x\) and \(y = -3y + 4\) that is \(x = 0\) and \(y = 1,\) or \(z = j.\)

5. We have \(3e^{j\pi/3} = 3(\cos \pi/3 + j\sin \pi/3) = 3(\frac{1}{2} + \frac{\sqrt{3}}{2}j) = \frac{3}{2} + \frac{3\sqrt{3}}{2}j.\)

6. We first have \(3 + 4j = 5\arctan \frac{4}{3}\) so that \((3 + 4j)^5 = 5^5\arctan \frac{4}{3}\) = \(3125\angle 1.495.\)

7. We have
   \[
   \cos 3\theta + j\sin 3\theta = (\cos \theta + j\sin \theta)^3 = \cos^3 \theta + 3j\cos^2 \theta \sin \theta - 3\cos \theta \sin^2 \theta - j\sin^3 \theta.
   \]
   Equating real parts we have \(\cos 3\theta = \cos^3 \theta - 3\cos \theta \sin^2 \theta.\) Various alternative forms are possible. For example, \(\cos 3\theta = \cos^3 \theta - 3\cos \theta(1 - \cos^2 \theta) = 4\cos^3 \theta - 3\cos \theta.\)

8. With \(z = \cos \theta + j\sin \theta\) we have
   \[
   (2 \cos \theta)^3 = (z + 1/z)^3 = z^3 + 3z + 3/z + 1/z^3 = (z^3 + 1/z^3) + 3(z + 1/z).
   \]
   Thus, \(\cos^3 \theta = \frac{1}{4} \cos 3\theta + \frac{3}{4} \cos \theta.\)

9. With \(w = z^2\) the equation \(z^4 - 2z^2 + 10 = 0\) becomes \(w^2 - 2w + 10 = 0\) so \(w = 1 \pm 3j.\) In polar form \(w = \sqrt{10\angle (\arctan 3)}\). The required \(z\) \((z = w^{1/2})\) values are then \(10^{1/4}\angle (\pm \frac{1}{2} \arctan 3)\) and \(10^{1/4}\angle (\pi \pm \frac{1}{2} \arctan 3).\)
10. In polar form $1 + j = \sqrt{2} \angle (\pi/4)$ so $(1 + j)^{1/3} = 2^{1/6} \angle (\pi/4 + 2n\pi)/3$. The principle values of the three roots are obtained by setting $n = -1, 0, 1$.

11. Using the identity
$$\sinh(A + B) = \cosh A \sinh B + \sinh A \cosh B$$
we have
$$\sinh(2x + jy) = \cosh 2x \sinh jy + \sinh 2x \cosh jy$$
$$= \cosh 2x (j \sin y) + \sinh 2x \cos y \quad .$$

Next, we use the identities
$$\cosh 2A = \cosh^2 A + \sinh^2 B$$
and
$$\sinh 2A = 2 \cosh A \sinh A$$
to write
$$\sinh(2x + jy) = j(\cosh^2 x + \sinh^2 x) \sin y + (2 \cosh x \sin x) \cos y.$$  

12. If $z = x + jy$ ($x$ and $y$ real) then
$$4 = \sin z$$
$$= \sin(x + jy)$$
$$= \cos x \sin(jy) + \sin x \cos(jy)$$
$$= \cos x (j \sin y) + \sin x \cosh y$$

Thus, equating real and imaginary parts, we have
$$\sin x \cosh y = 4 \quad \text{and} \quad \cos x \sinh y = 0.$$ 

The equation $\cos x \sinh y = 0$ tells us that either $\cos x = 0$ or $\sinh y = 0$, that is either $x$ is an odd multiple of $\pi/2$ or $y = 0$.

The equation $\sin x \cosh y = 4$ with $y = 0$ gives $\sin x = 4$ which is clearly invalid. We must then have $x$ an odd multiple of $\pi/2$. However, as $\cosh y$ is positive, $\sin x$ must also be positive and so we have $x = 2n\pi + \pi/2$ where $n$ is an integer. In this case $\sin x = 1$ and so $\cosh y = 4$ or
$$y = \ln(4 + \sqrt{16 - 1}) = \ln 7.873 = 2.063.$$ 

We thus have $x = 2n\pi + \pi/2$ where $n$ is an integer and $y = 2.063$.

13. In general we have $\ln z = \ln |z| + j \arg z$. Thus with $\ln z = 3 + j \frac{\pi}{3}$, we have $\ln |z| = 3$, or $|z| = e^3$, and $\arg z = \pi/3$. Thus $z = e^{3(\cos(\pi/3) + j \sin(\pi/3))} = e^{3(\frac{1}{2} + j \frac{\sqrt{3}}{2})} = 10.04 + 17.39j$. 