SOLUTIONS TO TUTORIAL SHEET 1

QUESTION 1 The solutions to these inequality type problems can be done a number of ways. Before solving the problems in a mathematically and logic manner, some comments on how they relate to graphs. Consider the first inequality statement

\[ 1 \leq x + 2 \leq 4, \]

which is essential two inequalities, namely

\[ 1 \leq x + 2 \quad \text{and} \quad x + 2 \leq 4. \]

If we graph the function \( f : \mathbb{R} \to \mathbb{R} \) defined by

\[ f(x) = x + 2, \]

then the question asks us to find the set, call it \( D \), of values \( x \) in the domain of \( f \), such that the range of the function for \( x \in D \) is in the interval \([1, 4]\). The graph is very simple to sketch, it being linear and continuously increasing. Use a graphics calculator or capgraph or any other graphing program you may have to investigate a solution to this problem.

We can analyse graphically the remaining parts of Question 1. For example in (ii), we can consider two functions \( f_1 : \mathbb{R} \to \mathbb{R} \), and \( f_2 : \mathbb{R} \to \mathbb{R} \), defined by

\[ f_1(x) = |x - 2| \quad \text{and} \quad f_2(x) = x. \]

Both functions have the same domain, that is, \( \text{dom}(f_1) = \text{dom}(f_2) = \mathbb{R} \). The question asks to find the set of \( x \) in the common domain for which

\[ f_1(x) < f_2(x). \]

One can also for this question consider the function \( f_3 = f_1 - f_2 : \mathbb{R} \to \mathbb{R} \) defined by

\[ f_3(x) = f_1(x) - f_2(x) = |x - 2| - x. \]

The question can then asks to find the set of \( x \) in the domain of \( f_3 \) for which the function’s values are strictly less than zero.

It is left for you to examine the remaining parts of this Questions using a graphical analysis. Some preliminary comments before we proceed further.

Given statements \( A \) and \( B \), if given \( A \) is true we can deduce that \( B \) is true, we say that the truth of \( A \) is necessary for the truth of \( B \), or the truth of \( A \) necessarily implies the truth of \( B \), and write

\[ A \Rightarrow B. \]
So for example take $A : x < (2 - x)$ and $B : x < 1$. Given $A$ is true for a value of $x$ then since
\[ x < (2 - x) \Rightarrow 2x < 2 \] (adding $x$ to both sides)
and
\[ 2x < 2 \Rightarrow x < 1 \] (dividing both sides by positive 2),
this implies that statement $B$ is true for this value of $x$. It should be clear that we can reverse the above argument and deduce that if $B$ is true for a given $x$ then $A$ is also true for the same $x$. That is, the truth of $B$ is sufficient for the truth of $A$. With implication both ways we write $A \Rightarrow B$ and say that the true of $A$ is necessary and sufficient for the truth of $B$, or $A$ is true if and only if $B$ is true. Sometimes it is said that statement $A$ is equivalent to statement $B$, but this is verging on other notation and characteristics of equivalence relations, to be learnt typically in second year.

Be careful and note: If we take $A : x^{1/2} < 1$ and $B : x < 1$ then it should be clear that the truth of $A$ implies the truth of $B$ since squaring both sides of the inequality for $A$ we achieve the inequality in $B$. Thus the truth of $A$ is necessary for the truth of $B$. However the truth of $B$ is not sufficient for the truth of $A$, clearly the set of $x$ for which $B$ is true contains negative numbers for which we cannot take the square root.

(i) As explained above the given inequality
\[ 1 \leq x + 2 \leq 4, \]
is essentially two inequalities, namely
\[ 1 \leq x + 2 \quad \text{AND} \quad x + 2 \leq 4. \]
Now the inequality $1 \leq x + 2$ is true if and only if $-1 < x$, that is, $x \in (-1, \infty)$. Similarly $x + 2 < 4$ is true if and only if $x < 2$, that is , $x \in (-\infty, 2)$.

Hence the given inequality must be true for the set of $x$ common to both these regions, namely their intersection, for which the two separate inequalities hold, that is,
\[ x \in (-1, \infty) \cap (-\infty, 2) = (-1, 2). \]

(ii) We are required to find the set of $x$ for which the inequality $|x - 2| < x$ is true. In this problem we have to come to grips with handling the absolute value function. It is defined by $\text{abs} : \mathbb{R} \to \mathbb{R}^+$, and
\[ y = \text{abs}(x) = |x| = \begin{cases} x & \text{if } x \geq 0, \\ -x & \text{if } x < 0, \end{cases} \]
Since we are considering values $|x - 2|$, it is necessary to consider $x$ values in two regions; $x < 2$ for which the argument is negative, and $x > 2$ for which the argument is positive. We consider $x = 2$ as a special case.

CASE: $x < 2$.

For $x$ in this region, the inequality is now defined as $-(x - 2) < x$. This inequality is true if and only if $2 < 2x$ or $1 < x$ is true. We can now state that the given inequality is satisfied only for $x \in (1, 2)$ for this case.
CASE: $x > 2$.
For $x$ in this region, the inequality is now defined as $(x - 2) < x$. Now this inequality is true if and only if $-2 < 0$ is true. Since it is indeed true that $-2 < 0$, then the given inequality must be satisfied for all $x \in (2, \infty)$ in this case.

CASE: $x = 2$.
Since by evaluation of the inequality, $0 < 2$ is indeed true, then the inequality is satisfied at $x = 2$.

In summary, the inequality is satisfied for all $x \in (1, 2) \cup \{2\} \cup (2, \infty) = (1, \infty)$.

(iii) We are to find the values of $x$ for which the following inequality is satisfied:

$$|2x| > |5 - 2x|.$$  

Using the fact that $|ab| = |a||b|$ is true for any two real numbers, $a$ and $b$, the problem can be simplified as follows:

$$|2x| > |5 - 2x|$$ is true if and only if $2|x| > 2|5/2 - x|$ is true,

and

$$2|x| > 2|5/2 - x|$$ is true if and only if $|x| > |5/2 - x|$ is true,

We are now to find the values of $x$ for which $|x| > |5/2 - x|$ is satisfied.

In this problem, we now have to consider two absolute functions in the definition of the inequality. The first absolute value, $|x|$ requires different evaluation depending upon whether $x \leq 0$ and $x \geq 0$. The second requires different evaluation depending upon whether $x \leq 5/2$ and $x \geq 5/2$. Considering all values on the real line, we need to consider then the following special cases:

CASE $x < 0$.
For $x$ in this specified region, the argument of $|x|$ is negative and the argument of $|5/2 - x|$ is positive. Hence the given inequality can now be written:

$$-x > (5/2 - x).$$

This inequality is true for a given $x$ if and only if $0 > 5/2$ is true. But clearly $0 > 5/2$ is not true. We conclude that the inequality cannot be satisfied for any $x < 0$.

CASE $0 < x < 5/2$.
For $x$ in this specified region, the argument of $|x|$ is positive and the argument of $|5/2 - x|$ is positive. Hence the given inequality can now be written:

$$x > (5/2 - x).$$

This inequality is true for a given $x$ if and only if $2x > 5/2$ or $x > 5/4$ is true. We conclude that in this case the inequality is satisfied for $x \in (5/4, 5/2)$. 

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CASE $5/2 < x$.

For $x$ in this specified region, the argument of $|x|$ is positive and the argument of $|5/2 - x|$ is negative. Hence the given inequality can now be written:

$$x > -(5/2 - x).$$

This inequality is true for a given $x$ if and only if $0 > -5/2$ is true. As this inequality is always true, we conclude that in this case the inequality is satisfied for $x \in (5/2, \infty)$.

CASE $x = 0$.

Since by direct evaluation $0 > 5/2$ is not true, the given inequality is not satisfied for $x = 0$.

CASE $x = 5/2$.

By direct evaluation, the inequality states that $5/2 > 0$, which is always true. Hence it is satisfied for $x = 5/2$.

In summary the given inequality is satisfied for all $x \in (5/4, 5/2) \cup \{5/2\} \cup (5/2, \infty) = (5/4, \infty)$.

(iv) We are to find the values of $x$ for which the following inequality is satisfied:

$$x^2 - 2x + 5 < 20.$$

We can rewrite this inequality simply as

$$x^2 - 2x - 15 < 0.$$

The question asks us to find those $x$ in the domain of the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2 - 2x - 15$, at which $f(x) < 0$. It is easy to see that this quadratic function factorises as $f(x) = (x + 3)(x - 5)$, and that $f(x) = 0$, when $x = -3$ and $x = 5$. From the graph we know then that $f(x) < 0$, when $x \in (-3, 5)$.

Consider the inequality as $(x + 3)(x - 5) < 0$. Now the product of two real numbers can only be negative if one is positive and the other negative. So to determine which $x$ satisfy the inequality we need to examine the regions on the real line for which the two factors have positive and negative sign.

CASE $x < -3$.

In this region factor $(x + 3)$ is negative and $(x - 5)$ is also negative, so their product is positive. Hence no $x$ in this region satisfy the given inequality.

CASE $-3 < x < 5$.

In this region factor $(x + 3)$ is positive and $(x - 5)$ is also negative, so their product is negative. Hence all $x$ in this region satisfy the given inequality, that is, $x \in (-3, 5)$.

CASE $x < -3$.

In this region factor $(x + 3)$ is positive and $(x - 5)$ is also positive, so their product is positive. Hence no $x$ in this region satisfy the given inequality.

Clearly values $x = -3$ and $x = 5$ do not satisfy the inequality, so the only $x$ which satisfy the inequality are $x \in (-3, 5)$. 

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(v) We are to find the values of $x$ for which the following inequality is satisfied:

$$\frac{2}{7 - 3x} \leq -5.$$ 

The answer to this question involves recognising that we must consider two cases: (a) when $(7 - 3x) > 0$ and (b) when $(7 - 3x) < 0$. This is because if we multiply across the inequality by the factor $(7 - 3x)$ when it is negative, then we must reverse the inequality.

CASE (a) $(7 - 3x) > 0$ or $x < 7/3$.

In this case the set of $x$ for which the inequality is satisfied is the same set for which $2 \leq -5(7 - 3x)$ is satisfied. Now this inequality is equivalent to $37 \leq 15x$ or $37/15 \leq x$. As $37/15 > 7/3$, no $x$ in this case satisfies the inequality.

CASE (b) $(7 - 3x) < 0$ or $x > 7/3$.

In this case the set of $x$ for which the inequality is satisfied is the same set for which $2 \geq -5(7 - 3x)$ is satisfied. Now this inequality is equivalent to $37 \geq 15x$ or $37/15 \geq x$. As $37/15 > 7/3$, all $x \in (7/3, 37/15]$ satisfy the inequality.

In summary the given inequality is satisfied for $x \in (7/3, 37/15]$.

(vi) We are to find the values of $x$ for which the following inequality is satisfied:

$$(x + 2)(2x - 1)(3x + 7) \geq 0.$$ 

In this problem we have to consider the sign of three factors on the left hand side. Clearly the inequality is satisfied for the three values $x = -2, x = 1/2$ and $x = -7/3$. The given inequality is equivalent to

$$(x + 2)(x - 1/2)(x + 7/3) \geq 0.$$ 

We need to consider 4 separate regions on the real line.

CASE $x < -7/3$.

In this case factor $(x + 2)$ is negative, $(x - 1/2)$ is negative and $(x + 7/3)$ is negative. Hence the product of these three factors is negative for any $x$ in this region. We conclude then no $x$ in this region satisfies the given inequality.

CASE $-7/3 < x < -2$.

In this case factor $(x + 2)$ is negative, $(x - 1/2)$ is negative and $(x + 7/3)$ is positive. Hence the product of these three factors is positive for any $x$ in this region. We conclude that for all $x \in (-7/3, -2)$ in this region satisfies the given inequality.

CASE $-2 < x < 1/2$.

In this case factor $(x + 2)$ is positive, $(x - 1/2)$ is negative and $(x + 7/3)$ is positive. Hence the product of these three factors is negative for any $x$ in this region. We conclude then no $x$ in this region satisfies the given inequality.

CASE $1/2 < x$.

In this case factor $(x + 2)$ is positive, $(x - 1/2)$ is positive and $(x + 7/3)$ is positive. Hence the product of these three factors is positive for any $x$ in this region. We conclude that for all $x \in (1/2, \infty)$ in this region satisfies the given inequality.
In summary the given inequality is satisfied for \( x \in (-7/3, -2) \cup (1/2, \infty) \). Note the answer here is a disjoint union of two intervals. Check this result graphically, by plotting the cubic polynomial on the left hand side of the inequality.

**QUESTION 2**

We are to show that

\[
\frac{|x - 2|}{x^2 + 9} \leq \frac{|x| + 2}{9} \quad \text{for all } x.
\]

Since the quantities \( x^2 + 9 \) and \( 9 \) on the left and right hand side of the inequality are always positive, then the inequality is equivalent to

\[9|x - 2| \leq (x^2 + 9)(|x| + 2)\]

Since we need to consider the sign of the arguments of the two absolute value functions in this inequality, there are three cases to consider on the real line.

**CASE** \( x < 0 \).

In this case, using the definition of the absolute value function \( |x - 2| = -(x - 2) \), and \( |x| = -x \), and the inequality becomes:

\[-9(x - 2) \leq (x^2 + 9)(-x + 2) = -(x^2 + 9)(x - 2).
\]

Since \( (x - 2) < 0 \) here and \( -(x - 2) > 0 \), this inequality is equivalent to

\[9 \leq (x^2 + 9).
\]

As this inequality is always true, then the given inequality is satisfied for all \( x \) in this region.

**CASE** \( 0 < x < 2 \).

In this case, using the definition of the absolute value function \( |x - 2| = -(x - 2) \), and \( |x| = x \), and the inequality becomes:

\[-9(x - 2) \leq (x^2 + 9)(x + 2) = (x^2 + 9)(x + 2) = x^3 + 2x^2 + 9x + 18.
\]

This inequality is equivalent to

\[0 \leq x^3 + 2x^2 + 18x.
\]

This last inequality is true for all \( x \) in this region for \( x > 0 \).

**CASE** \( 2 < x \).

In this case, using the definition of the absolute value function \( |x - 2| = (x - 2) \), and \( |x| = x \), and the inequality becomes:

\[9(x - 2) \leq (x^2 + 9)(x + 2) = (x^2 + 9)(x + 2) = x^3 + 2x^2 + 9x + 18.
\]

This inequality is equivalent to

\[0 \leq x^3 + 2x^2 + 36.
\]

This last inequality is true for all \( x \) in this region for \( x > 0 \).

The inequality is also satisfied at \( x = 2 \) and \( x = 0 \). Check this. In summary we conclude the given inequality is satisfied for all real \( x \).
QUESTION 3

The graphs of the six functions are shown in the Figures below, they were obtained using the Capgraph program. From them, we deduce the following information as required:

(a) Given \( f(x) = 2x - 3 \), the \( \text{dom}(f) = \text{range}(f) = \mathbb{R} \), that is, the domain and range of \( f \) equal the real line. Clearly the function is monotonic (strictly) increasing over this domain. The graph is simply a straight line graph with slope 2 and intercept \(-3\).

(b) Given \( f(x) = 3x^2 - 1 \), the \( \text{dom}(f) = \mathbb{R} \), and \( \text{range}(f) = [-1, \infty) \). The function is monotonic decreasing on \((-\infty, 0)\) and monotonic increasing on \((0, \infty)\).

(c) Given \( f(x) = x^3 - x = x(x^2 - 1) \). From the graph we see that \( \text{dom}(f) = \text{range}(f) = \mathbb{R} \). Using Capgraph we can find approximate intervals for which the function is monotonic increasing and decreasing. Exact results are: the function in monotonic increasing on the
intervals \((-\infty, -\frac{1}{\sqrt{3}})\) and \((1/\sqrt{3}, \infty)\), and the function is monotonic decreasing on the interval \((-\frac{1}{\sqrt{3}}, 1/\sqrt{3})\).

(d) Given \(f(x) = \frac{x^2 + 3x}{x + 3}\). Observe this function is NOT defined at \(x = -3\). Hence the \(\text{dom}(f) = (-\infty, -3) \cup (-3, \infty)\). For all other values of \(x\) in the domain, we find that:

\[
f(x) = \frac{x^2 + 3x}{x + 3} = x(x + 3)/(x + 3) = x.
\]

Excluding the point \((-3, -3)\) the graph of \(f\) is simply the straight line graph with slope 1 and intercept 0. Hence the \(\text{range}(f) = (-\infty, -3) \cup (-3, \infty)\) also. The function is clearly monotonic (strictly) increasing on its domain of definition.

(e) Given \(f(x) = (x^2 - 3x + 2)^{1/2} = ((x - 1)(x - 2))^{1/2}\). Observe this function is only defined provided the argument of the square root function is non-negative. This argument \(x^2 - 3x + 2 = (x - 1)(x - 2)\) is a simple quadratic function of \(x\) whose range values \(f(x) < 0\) if \(x \in (1, 2)\), and \(f(x) \geq 0\) if \(x \in (-\infty, 1] \cup [2, \infty)\). Hence the \(\text{dom}(f) = (-\infty, 1] \cup [2, \infty)\), and since the values of the square root must only be positive by definition of the function, the \(\text{range}(f) = [0, \infty)\). The function is monotonic decreasing on the interval \((-\infty, 1)\) and monotonic increasing on the interval \((2, \infty)\).

NOTE: Remember the square root function is a function, it is single valued and \(x^{1/2} \geq 0\) for all \(x \geq 0\).

(f) Given \(f(x) = (x^3 - x^2 - |x^3 - x^2|)/2\). Using the definition of the absolute value function

\[
|x^3 - x^2| = \begin{cases} x^3 - x^2 & \text{if } x^3 - x^2 \geq 0, \\ -(x^3 - x^2) & \text{if } x^3 - x^2 < 0,
\end{cases}
\]

we can write that

\[
f(x) = \begin{cases} 0 & \text{if } x^3 - x^2 \geq 0, \\ (x^3 - x^2) & \text{if } x^3 - x^2 < 0.
\end{cases}
\]

Clearly the \(\text{dom}(f) = \mathbb{R}\), and the \(\text{range}(f) = (-\infty, 0]\). The function is monotonic increasing on the intervals \((-\infty, 0)\) and \((2/3, 1]\). It is monotonic decreasing on the interval \((0, 2/3)\).
**QUESTION 4**

We consider the discrete function defined on the non-negative integers defined by:

\[ f(n) = \frac{10^n}{n!}. \]

This exercise is one of using a calculator to examine the behaviour of this discrete function. Values obtained for \( n = 0 \) to \( n = 41 \) are tabulated below! Note the factorial function defined for natural numbers is \( n! = n(n-1)(n-2) \cdots (2)1 \), so for example, \( 5! = (5)(4)(3)(2)1 = 120 \).

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(a) We observe that the function values \( f(n) \) → 0 as \( n \to \infty \). This means that the factorial function \( n! \) increases to \( \infty \) at a rate much faster than \( 10^n \).

(b) The function is neither monotonic (strictly) increasing or decreasing as can be seen from the graph of the function, see Figure 4.

(c) Examining the table and the graph of the function, it is clearly monotonic decreasing for \( n > 10 \). Can you see by writing out the individual terms for each \( n \) why this is so?

(d) Also examining the table we find that for \( N = 29 \), \( |f(n)| < 0.01 \) for all \( n > N \).

**QUESTION 5**

We are given function \( f : \mathbb{R}^+ \to \mathbb{R} \), where \( \mathbb{R}^+ \) is the set of all positive real numbers. It is assumed that the following identity

\[ f(xy) = f(x) + f(y), \]
holds for all $x, y > 0$.

(a) Let $x = 1$ and $y = 1$, then the defining equality implies

$$f(1) = f(1) + f(1) = 2f(1).$$

Hence we conclude that $f(1) = 0$.

(b) Assuming there is a number $A$ such that $f(A) = 1$. Then setting $x = A$ and $y = A$ in the defining equality implies

$$f(A^2) = f(A) + f(A) = 2.$$ 

Now set $x = A$ and $y = 1/A$, then as above

$$f((A)(1/A)) = f(1) = f(A) + f(1/A) = 1 + f(1/A).$$

From this we conclude that $f(1/A) = -1$. If we set $x = A^2$ and $y = A$, then as above

$$f(A^3) = f(A^2) + f(A) = 2 + 1 = 3.$$ 

Clearly we can continue the process realising $f(A^n) = n$. It can be formally shown using the method of Mathematical Induction.

(c) Let $x = 0$ and $y = A$, then the defining equality states the function satisfies

$$f((0)(A)) = f(0) = f(0) + f(A) = f(0) + 1.$$ 

There is clearly no finite value that we can attach to $f(0)$ that will be consistent with this equality.

**QUESTION 6**

We are asked to find the range of the function $f : R \to R$ defined by:

$$y = f(x) = \begin{cases} 
-1/x^2 & \text{if } x \leq -1, \\
17 & \text{if } -1 < x < 1, \\
1/x^2 & \text{if } 1 \leq x. 
\end{cases}$$
For values $x$ in the subdomain $(-\infty, -1]$, the range of values for $-1/x^2$ lie in the interval $[-1, 0)$. Note that the value 0 is never attained for $-1/x^2 \to 0$ as $x \to -\infty$.

For values $x$ in the subdomain $(-1, 1)$, the range of values is just the set $\{17\}$ since the functions value in this interval is constant.

For values $x$ in the subdomain $[1, \infty)$, the range of values for $1/x^2$ lie in the interval $(0, 1]$. Again the value 0 is never attained for $1/x^2 \to 0$ as $x \to \infty$.

Hence the range of $f$, range$(f) = [-1, 0) \cup \{17\} \cup (0, 1]$.

**QUESTION 7**

We seek a relation between $V$ and $P$ of the form $P = kV^m$, for constants $k$ and $m$.

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</table>

For the first two pieces of data, the formula determines the following equations hold:

\[ 28.3 = k(1.46)^m \quad \text{and} \quad 13.3 = k(2.50)^m. \]
From these by division we obtain an equation for the unknown $m$:

$$\frac{28.3}{13.3} = \left(\frac{1.46}{2.50}\right)^m.$$  

Taking the natural logarithm on both sides:

$$\ln\left(\frac{28.3}{13.3}\right) = m \ln\left(\frac{1.46}{2.50}\right).$$

From this we find $m = -1.40391$. Using the first equation we solve to find $k = 48.141881$. Hence the given relationship between pressure and volume is

$$P = \frac{48.141881}{V^{1.40391}}.$$  

Note that if we had taken the natural logarithm of $P = kV^m$ initially, then

$$\ln(P) = \ln(k) + m \ln(V),$$

which expresses a linear relationship between \(\ln(P)\) and \(\ln(V)\), with slope $m$ and intercept $\ln(k)$.

Check that the other values in the table satisfy approximately the determined relationship. This relationship between pressure and volume is know as Boyle’s law.