Question 1

(a) In physiology, the Dubois formula relates a person’s surface area, \( S \) in \( m^2 \), to weight, \( W \) in \( kg \), and height \( H \) in \( cm \), by

\[
S = 0.01W^{0.25}H^{0.75}.
\]

(i) What is the surface area of a person who weighs 65 kg and is 160 cm tall?

(ii) What is the weight of a person whose height is 180 cm and who has a surface area of 1.5 \( m^2 \)?

(iii) For people of fixed weight 70 kg, solve for \( H \) as a function of \( S \). Simplify your answer.

(b) A baseball hit at an angle \( \theta \) to the horizontal with initial velocity \( v_0 \) has a horizontal range, \( R \), given by

\[
R(\theta) = \frac{v_0^2}{g} \sin(2\theta).
\]

Here \( g = 9.81 m/s^2 \) is a constant, the acceleration due to gravity. Sketch roughly the graph of \( R \) as a function of \( \theta \) for \( 0 \leq \theta \leq \pi/2 \). What angle gives the maximum range? What is the maximum range?

(14, 6 marks)

Question 2

(a) Given the function \( f \) defined by the evaluation,

\[
f(x) = \frac{1}{\sqrt{x} + \sqrt{x}},
\]

express \( f \) as a composition of three functions defining for each their domain and range.

(b) An analysis of an ancient campfire shows that one-tenth of the carbon 14 in the original ashes has decomposed. Use this information to date the campsite. (The half-life of carbon 14 is 5568 years.)

Question 2 continued overleaf
Question 2 continued

(c) Suppose we are given the function \( f \) defined by the evaluation
\[
y = f(x) = \frac{x^2 - 1}{x^3}.
\]

(i) Determine all horizontal and vertical asymptotes.

(ii) Calculate the first derivative \( f' \) and determine the intervals upon which the given function is increasing and decreasing.

(iii) Is the function odd or even?

(iv) With this knowledge draw a rough hand graph of the function.

All working must be shown. (6, 6, 8 marks)

Question 3

(a) Find parametric equations for the path of a particle that moves counterclockwise halfway around the circle
\[
(x - 2)^2 + (y - 2)^2 = 4,
\]
from the bottom to the top.

(b) Find \( \frac{dy}{dx} \) when
\[
y = f(x) = \frac{x^{3/4} \sqrt{x^2 + 1}}{(3x + 2)^5} \quad \text{(HINT: Take logarithms)}
\]

(ii) \( x^3 + y^3 = 6xy \).

(c) Consider the function
\[
f(x) = x^4 - 2x^2.
\]

(i) Determine the local minimum and maximum points of \( f \).

(ii) Determine the intervals in which the function is concave up and concave down.

(iii) Use this information obtained to free-hand sketch the main features of the function. All working must be shown. (6, 6, 8 marks)

Question 4 overleaf
Question 4

(a) Consider the set of simultaneous equations:

\[
\begin{align*}
2x + 5y - 3z &= 2 \\
x + 3y - 2z &= 1 \\
-3x + 2y - 4z &= 1
\end{align*}
\]

with coefficient matrix

\[
A = \begin{bmatrix}
2 & 5 & -3 \\
1 & 3 & -2 \\
-3 & 2 & -4
\end{bmatrix}
\]

(i) Find the inverse of the matrix \(A\) by Gauss-Jordan reduction using elementary row operations. Verify that your answer is correct. **All working must be shown.**

(ii) Solve the system of equations using the inverse of \(A\). **All working must be shown.**

(b) Using the Gauss-Jordan method, solve the following linear system. If the system has no solutions, then state so. If the system has infinitely many solutions, express them parametrically. **All working must be shown.**

\[
\begin{align*}
x + 3y + 4z &= 1 \\
-2x - 5y - 3z &= -1 \\
x + 4y + 9z &= 2
\end{align*}
\]

(10, 10 marks)
Question 5

(a) Let $P$ be the point with position vector $\mathbf{v} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ relative to origin $O$. Consider the plane passing through the origin described by

$$6x + 2y - 3z = 0.$$ 

Determine $ON$ the magnitude of the projection $u$ of the line $OP$ on the plane as shown in the following figure. Vector $n$ is the unit normal to the given plane.

(b) Determine the Cartesian equation of the plane determined by the three points $(0,1,2)$, $(1,1,1)$ and $(-1,1,-1)$.

(c) Find the volume of the tetrahedron formed by the origin $O$ and the points $A$, $B$, and $C$ with position vectors $2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$, $3\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{i} + \mathbf{j} + \mathbf{k}$ respectively relative to the origin.

$(8, 6, 6$ marks$)$
Question 6

(a) Solve the following equation for the variable \( x \),

\[
\log(x^6) - \log(5x + 14) = 4 \log(x).
\]

(b) Use Newton’s method with \( x_1 = 1 \) to find the root of the equation \( x^3 - x = 1 \) correct to six (6) decimal places. **All working must be shown.**

(c) A rectangular sheet of metal of dimension 2 metres by 1.5 metres has a square portion of length \( x \) metres cut from each corner. The remaining metal is then bent to form an open rectangular box of depth \( x \) metres.

(i) What is the value of \( x \) that maximises the volume of the box?
(ii) What is the maximum volume of the box? **(5, 7, 8 marks)**

Question 7

(a) Find \( \frac{dy}{dx} \) when

(i) \( y = f(x) = \arctan(x^{1/2}) \).
(ii) \( y = f(x) = e^{-x} \sin(x) \).

(b) Use De Moivre’s theorem to express \( (2\sqrt{3} + 2j)^5 \) in simplified rectangular form.

(c) Using L’Hopital’s Rule evaluate the following limit:

\[
\lim_{x \to 1} \frac{1 - x + \ell n(x)}{1 + \cos(\pi x)}.
\]

(d) Given the function \( f \) defined by the evaluation

\[
f(x) = \frac{x + 1}{x - 1},
\]

calculate the derivative \( f' \) at \( x = 2 \). **(6, 5, 4, 5 marks)**