INSTRUCTIONS FOR STUDENTS

This examination paper comprises 66% of the total assessment for this course.

The examination comprises twelve questions. All questions are of equal value.

Attempt as many questions as possible. It is expected that eight questions can be completed in the time available.

All working must be shown to gain full marks.

All answers should be written in the examination answer booklets supplied to you.
Question 1

(a) Evaluate the expression, all numbers are approximate.

\[
\frac{8.896 \times 10^{-12}}{3.5954 + 6.0449}
\]  

(2 Marks)

(b) Solve the equation for \( x \).

\[
\frac{2(x - 4)}{3} = \frac{5}{4}
\]  

(2 Marks)

(c) Simplify \( \frac{-35x^{-1}y(x^2y)}{5xy^{-1}} \)  

(1 Mark)

(d) A carat equals \( \frac{1}{24} \) part of gold in an alloy (for example, 9-carat gold is \( \frac{9}{24} \) gold). How many grams of 9-karat gold must be mixed with 18-carat gold to get 180g of 14-carat gold?  

(3 Marks)
Question 2

(a) Determine the indicated angles.

\[
\begin{align*}
\text{C} & \quad \text{D} \\
\text{A} & \quad \text{B} \\
52^\circ & \\
1 & \\
2 & \\
\end{align*}
\]

(i) \( \angle 1 \) \\
(ii) \( \angle 2 \) \hspace{1cm} (2 Marks)

(b) Given that \( AB = 8 \), \( BC = 8 \), \( CD = 12 \), and \( \angle ADC = 53^\circ \), find the following angle and lengths.

\[
\begin{align*}
\text{A} & \quad \text{B} \\
\text{C} & \quad \text{D} \\
\text{E} & \\
\end{align*}
\]

(i) \( \angle ABE \) \\
(ii) \( AD \) \\
(iii) \( BE \) \\
(iv) \( AE \) \hspace{1cm} (4 Marks)

(c) A tent is in the shape of a regular pyramid surmounted on a cube. If the edge of the cube is 2.50m and the total height of the tent is 3.25m, find the area of the material used in making the tent (not including any floor area). \hspace{1cm} (2 Marks)


**Question 3**

(a) Determine the domain and the range of the function \( g(t) = \frac{t - 0.087629}{3.0125t} \), find \( g(8.91) \) and \( g(-4.91) \). These values of \( t \) are approximate. (2 Marks)

(b) The voltage \( V \) and current \( I \) (in mA) for a certain electrical experiment were measured as shown in the following table. Plot the graph of \( i = f(V) \) and from the graph determine \( f(4.5) \).

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<th>Voltage (V)</th>
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<th>3.0</th>
<th>4.0</th>
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(3 Marks)

(c) One ball bearing is 1.00mm more in radius and has twice the volume of another ball bearing. What is the radius of each? (3 Marks)

**Question 4**

(a) Solve the equation by using the quadratic formula.

\[ 12.5R^2 + 13.2R = 15.5 \]

(2 Marks)

(b) Solve the following equation by any method. All numbers are accurate to at least two significant digits.

\[ \frac{5}{G} + 1 = \frac{G-1}{3} \]

(3 Marks)

(c) If \( y = x^2 - 8x + 8 \), sketch the graph using the vertex and any other useful points and find graphically the values of \( x \) for which \( y = 0 \). (3 Marks)
Question 5

(a) Determine the centre and radius of the circle given by the equation
\[ x^2 + y^2 + 22x + 14y = 26 \]  
(4 Marks)

(b) An airplane consumes 140L of fuel in flying 680km. Under similar conditions, how far can it fly on 240L?  
(2 Marks)

(c) Find v when r = 2, s = 3 and t = 4 if v varies jointly as r and s and inversely as the square of t and v = 8 when r = 2, s = 6 and t = 6.  
(2 Marks)

Question 6

(a) Evaluate
\[ \sqrt{\frac{2 \ln 0.9523}{\log 6066}} - \sqrt{\log 5} \].  
(1 Mark)

(b) A projection of the annual growth rate p (in %) of the number of users of the internet is
\[ p = 8.5 \left(1.2^{-t} + 1\right) \], where t is the number of years after 2005. Draw the graph of this function on graph paper from 2005 to 2015.  
(5 Marks)

(c) If \( A_0 \) dollars are invested at 8%, compounded continuously for t years, the value \( A \) of the investment is given by
\[ A = A_0 e^{0.08t} \].

Determine how long it takes for the investment to double in value.  
(2 Marks)

Question 7

(a) The vertical displacement of a point at the end of a rotating wheel is
\[ y = 7.0 \sin \left(\frac{\pi t}{3}\right) \]. Sketch two cycles of y (in cm) as a function of t (in seconds) starting from \( t = 0 \).  
(3 Marks)

(b) A rocket is launched at an angle of 40.0º with the horizontal and with a speed of 860 m/s. What are the horizontal and vertical components of its velocity?  
(2 Marks)

(c) The propeller of the motor on a motorboat is rotating at 130 rad/s. What is the linear velocity of a point on the tip of a blade if it is 22.5cm long?  
(3 Marks)
**Question 8**

(a) Perform the following matrix multiplication:

\[
\begin{bmatrix}
-1 & 7 \\
2 & 0 \\
4 & -1
\end{bmatrix}
\begin{bmatrix}
1 & -4 & 5 \\
5 & 1 & 0
\end{bmatrix}
\]

(3 Marks)

(b) To find the electric currents (in amps), it is necessary to solve the following equations:

\[-I_A - 2 I_B = 0 \\
I_A - I_B = -6\]

Find \(I_A\) and \(I_B\) using any matrix method. (5 Marks)

**Question 9**

(a) In a road accident a car hit a safety barrier at an acute angle of \(\frac{\pi}{10}\). If the speed of the car was 70 km/h, what was the component of the speed perpendicular to the surface? (2 Marks)

(b) Two ships are being observed at the same observing station. One is 12 km from the station, and the other is 10 km away. The angle between their lines of observation is \(\frac{5\pi}{6}\). How far apart are the ships? (3 Marks)

(c) A weather balloon rises vertically at 4.5 m/s as the wind carries it horizontally at 2.5 m/s. What is the resultant velocity of the balloon? (3 Marks)

**Question 10**

(a) Find the derivative for: \(y = 7x^8 - 3^7 - x^5\). (1 Mark)

(b) Find the equation of a line tangent to the curve of \(y = 8x^3 - x^4\) at (1, 7). (4 Marks)

(c) A race track 400m long is to be built around an area that is a rectangle with a semicircle at each end. Find the open side of the rectangle (length of the straight edge of the track) if the area of the rectangle is to be maximised. (3 Marks)
### Question 11

(a) The altitude $h$ (in metres) of a ball thrown into the air as a function of the time $t$ (in seconds) is given by $h = 2 + 5t - t^2$. What is the maximum altitude that the ball attains? \hspace{1cm} (2 Marks)

(b) A `u'-shaped gutter is to be made by folding a piece of sheet metal that is 200cm wide so that the cross-sectional profile forms three sides of a rectangle. What is the maximum cross-sectional area of the gutter? \hspace{1cm} (4 Marks)

(c) Evaluate:
   
   (i) $\int (5x^4 - x^{-3}) \, dx$  
   (ii) $\int x^3 (x - 3x^4) \, dx$ \hspace{1cm} (2 Marks)

### Question 12

The home scores of the Bushmen Basketball team for the last season are:

84, 88, 77, 90, 92, 85, 99, 81, 92, 102, 92, 104

(a) Find the mean. \hspace{1cm} (2 Marks)

(b) Find the median. \hspace{1cm} (2 Marks)

(c) Find the standard deviation. \hspace{1cm} (2 Marks)

(d) Find the mode. \hspace{1cm} (2 Marks)
### Areas

- **Triangle**, $\frac{1}{2} \text{ (base} \times \text{perpendicular height)}$
- **Circle**, $\pi r^2$ (r is the radius)
- **Sector of Circle**, $\frac{r^2}{2} \theta$ ($\theta$ in radians)

### Trigonometry

- $2\pi$ radians is equivalent to 360°
- Length of arc of circle = $r\theta$
- For the functions, $f(x) = a \sin(bx + c)$, the period is $\frac{2\pi}{b}$, and the phase shift is $-\frac{c}{b}$

### Quadratic Functions

For $f(x) = ax^2 + bx + c$

The solution of $f(x) = 0$ is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, providing $b^2 \geq 4ac$

The equation of the axis of symmetry is $x = \frac{-b}{2a}$

### Rules of Logarithms

$\log_a MN = \log_a M + \log_a N$

$\log_a \left(\frac{M}{N}\right) = \log_a M - \log_a N$

$\log_a M^x = x \log_a M$

### Circles

The equation of a circle with centre $(a, b)$ and radius $r$ units is

$$(x - a)^2 + (y - b)^2 = r^2$$

### Rules of differentiation

- If $f(x) = ax^n$ then $f'(x) = nax^{n-1}$
- If $f(x) = u(x)v(x)$ then $f'(x) = u'(x)v(x) + u(x)v'(x)$
- If $f(x) = f(u)$ where $u = g(x)$ $f'(x) = \frac{df}{du} \cdot g'(x)$

### Rules of Integration

- If $f(x) = ax^n$, $n \neq -1$, $\int f(x)dx = \frac{ax^{n+1}}{n+1} + c$
- $\int \frac{dx}{x} = \ln x + c$
- If $\int g(x)dx = F(x)$ then $\frac{d}{dx} F(x) = g(x)$

### One method of finding the Determinant of a 3x3 Matrix:

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$
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