INSTRUCTIONS FOR STUDENTS

This examination paper comprises 66% of the total assessment for this course.

The examination comprises twelve questions. All questions are of equal value.

Attempt as many questions as possible. It is expected that eight questions can be completed in the time available.

All working must be shown to gain full marks.

All answers should be written in the examination answer booklets supplied to you.
Question 1

(a) Evaluate the expression, all numbers are approximate.
\[
\frac{1}{0.03568} + \frac{37466}{29.63^2}
\]
(2 Marks)

(b) Solve the equation for \(x\).
\[
0.250(6.721 - 2.44x) = 2.08
\]
(2 Marks)

(c) The time (in seconds) it takes a computer to check \(n\) memory cells is found by evaluating \(\left(\frac{n}{2650}\right)^2\). Find the time to check 48 cells. (1 Mark)

(d) Fifty kilograms of a cement-sand mixture is 40% sand. How many kilograms of sand must be added for the resulting mixture to be 60% sand? (3 Marks)

Question 2

(a) Determine the indicated angles.

(i) \(\angle CGE\)
(ii) \(\angle EGF\)
(iii) \(\angle DGH\)
(iv) \(\angle EGI\) (2 Marks)
(b) Given that $AB = 4$, $BC = 4$, $CD = 6$, and $\angle ADC = 53^\circ$, find the following angle and lengths.

(i) $\angle ABE$
(ii) $AD$
(iii) $BE$
(iv) $AE$

(4 Marks)

(c) Two people are talking to each other on cellular phones. If the angle between their signals at the tower is $90^\circ$, as shown in the diagram below, how far apart are they?

(2 Marks)

Question 3

(a) For the function $h(t) = \frac{t^2 - 4t}{t^3 + 564}$, find $h(8.91)$ and $h(-4.91)$. These values of $t$ are approximate. 

(2 Marks)
(b) In an experiment measuring the pressure \( p \) (in kPa) at a given depth of \( d \) (in metres) of seawater, the results in the following table were found. Plot the graph of \( p = f(d) \) and from the graph determine \( f(10) \).

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<th>6.0</th>
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<td>( p ) (kPa)</td>
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<td>161</td>
<td>193</td>
<td>225</td>
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(3 Marks)

(c) Two electrical resistors in parallel have a combined resistance \( R_f \) given by

\[
R_f = \frac{R_1R_2}{R_1 + R_2}.
\]

If \( R_2 = R_1 + 2.0 \), express \( R_f \) as a function of \( R_1 \) and find \( R_1 \) if \( R_f = 6.0\Omega \).

(3 Marks)

**Question 4**

(a) Solve the equation by using the quadratic formula.

\[0.30R^2 - 0.42R = 0.15\]

(2 Marks)

(b) Solve the following equation by any method. All numbers are accurate to at least two significant digits.

\[
\frac{20}{R} + \frac{20}{R+10} = \frac{1}{5}.
\]

(3 Marks)

(c) In an electric circuit, the voltage \( V \) as a function of the time \( t \) (in minutes) is given by \( V = 9.8 - 9.2t + 2.3t^2 \). Sketch the graph of \( V = f(t) \), for \( t \leq 5 \) minutes.

(3 Marks)

**Question 5**

(a) Given that 1.00 litre = 1000 cm\(^3\), what capacity in litres does a cubical box have that is 3.23 cm along an edge?  

(1 Mark)

(b) The dosage of a certain medicine is 25 ml for each 10 kg of the patient’s weight. What is the dosage for a person weighing 56 kg?  

(3 Marks)

(c) The period \( T \) of a pendulum varies directly as the square root of its length \( L \). If \( T = \pi / 2 \) seconds for \( L = 61.0 \text{ cm} \), find \( T \) for \( L = 122 \text{ cm} \).  

(4 Marks)
Question 6

(a) Evaluate $\frac{3\ln 12.5}{\sqrt{\log 7}}$. (1 Mark)

(b) The efficiency $e$ of a gasoline engine as a function of its compression ratio $r$ is given by $e = 1 - r^{1-y}$, where $y$ is a constant. Find $y$ for $e = 0.55$ and $r = 7.5$. (4 Marks)

(c) A computer analysis of the luminous efficiency $E$ of a tungsten lamp as a function of its input power $P$ (in Watts) is given by

$$E = 22.0 \left(1 - 0.65e^{-0.008P}\right).$$

Sketch the graph of $E$ as a function of $P$ for $0 \leq P \leq 1000$ Watts. (3 Marks)

Question 7

(a) The vertical displacement of a point at the end of a propeller blade of a small boat is $y = 14.0 \sin(40.0 \pi t)$. Sketch two cycles of $y$ (in cm) as a function of $t$ (in seconds) starting from $t = 0$. (3 Marks)

(b) A rocket is launched at an angle of $42.0^\circ$ with the horizontal and with a speed of 760 m/s. What are its horizontal and vertical components of velocity? (2 Marks)

(c) Solve the following equation for $x$ between $0$ and $2\pi$:

$$5 \sin x = 3 - (\sin x + 2).$$

(3 Marks)
**Question 8**

(a) Perform the following matrix multiplication:

\[
\begin{bmatrix}
-0.1 & 0.7 \\
0.2 & 0.0 \\
0.4 & -0.1
\end{bmatrix}
\begin{bmatrix}
0.1 & -0.4 & 0.5 \\
0.5 & 0.1 & 0.0
\end{bmatrix}
\]

(b) To find the electric currents (in amps), it is necessary to solve the following equations:

\[
\begin{align*}
I_A + I_B + I_C &= 0 \\
5I_A - 2I_B &= -4 \\
2I_B - I_C &= 0
\end{align*}
\]

Find \(I_A\), \(I_B\) and \(I_C\) using any matrix method.  

**(2 Marks)**  

**(6 Marks)**

**Question 9**

(a) In a ballistics test, a bullet was fired into a block of wood with a velocity of 670 m/s and at an angle of 71.3º with the surface of the block. What was the component of the velocity perpendicular to the surface?  

**(2 Marks)**

(b) Two satellites are being observed at the same observing station. One is 36 200 km from the station, and the other is 30 100 km away. The angle between their lines of observation is 105.4º. How far apart are the satellites?  

**(3 Marks)**

(c) A helium-filled balloon rises vertically at 3.5 m/s as the wind carries it horizontally at 5.0 m/s. What is the resultant velocity of the balloon?  

**(3 Marks)**

**Question 10**

(a) Find the derivative for: \(y = 8x^7 - 2^5 - x\).  

**(1 Mark)**

(b) Find the equation of a line tangent to the curve of \(y = 7x^4 - x^3\) at \((-1,8)\).  

**(4 Marks)**

(c) The displacement \(s\) (in cm) of a piston during each engine cycle is given by \(s = 8t - t^2\), where \(t\) is the time (in seconds). For what value(s) of \(t\) is the velocity of the piston 4 cm/s?  

**(3 Marks)**
Question 11

(a) The altitude $h$ (in metres) of a certain rocket as a function of the time $t$ (in seconds) after launching is given by $h = 550t - 4.9t^2$. What is the maximum altitude the rocket attains? (2 Marks)

(b) A special insulation strip is to be sealed completely around three edges of a rectangular solar panel. If 200 cm of the strip are used, what is the maximum area of the panel? (4 Marks)

(c) Evaluate:
   
   (i) $\int (4x^3 - x) \, dx$
   
   (ii) $\int x(x - 3x^4) \, dx$ (2 Marks)

Question 12

An important property of oil is the coefficient of viscosity, which gives a measure of how well it flows. In order to determine the viscosity of a certain motor oil, a refinery took samples from 12 different storage tanks and tested them at 50ºC. The results (in pascal-seconds) were 0.24, 0.28, 0.29, 0.26, 0.27, 0.26, 0.25, 0.27, 0.28, 0.26, 0.26, 0.25.

(a) Find the mean. (2 Marks)

(b) Find the median. (1 Mark)

(c) Find the standard deviation. (2 Marks)

(d) Draw a histogram. (3 Marks)
### Areas

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<th>Shape</th>
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<tr>
<td>Triangle</td>
<td>( \frac{1}{2} \text{ base \times \ perpendicular \ height} )</td>
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<tr>
<td>Circle</td>
<td>( \pi r^2 ) (( r ) is the radius)</td>
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<td>Sector of Circle</td>
<td>( \frac{r^2 \theta}{2} ) (( \theta ) in radians)</td>
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### Circles

The equation of a circle with centre \((a, b)\) and radius \(r\) units is
\[ (x-a)^2 + (y-b)^2 = r^2 \]

### Rules of differentiation

- If \( f(x) = a^x \) then \( f'(x) = n ax^{n-1} \)
- If \( f(x) = u(x)v(x) \) then \( f'(x) = u'(x)v(x) + u(x)v'(x) \)
- If \( f(x) = f(u) \) where \( u = g(x) \)
  \[ f'(x) = \frac{df}{du} \cdot g'(x) \]

### Rules of Integration

- If \( f(x) = ax^n, \ n \neq -1, \)
  \[ \int f(x)dx = \frac{ax^{n+1}}{n+1} + c \]
- \( \int \frac{dx}{x} = \ln x + c \)
- If \( \int g(x)dx = F(x) \) then \( \frac{d}{dx} F(x) = g(x) \)

### Quadratic Functions

For \( f(x) = ax^2 + bx + c \)

- The solution of \( f(x) = 0 \) is \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \), providing \( b^2 \geq 4ac \)
- The equation of the axis of symmetry is \( x = \frac{-b}{2a} \)

### Rules of Logarithms

\[
\log_a MN = \log_a M + \log_a N
\]
\[
\log_a \left( \frac{M}{N} \right) = \log_a M - \log_a N
\]
\[
\log_a M^x = x \log_a M
\]
### Table of Normal Probabilities

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